Comparing Multi-State Expected Damages, Option Price and Cumulative Prospect Measures for Valuing Flood Protection

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Abstract

Floods are risky events ranging from small to catastrophic. Although expected flood damages are frequently used for economic policy analysis, alternative measures such as option price and cumulative prospect value exist. The empirical magnitude of these measures whose theoretical preference is ambiguous is investigated using case study data from Baltimore City. The outcome for the base case option price measure increases mean willingness to pay over the expected damage value by about 3 percent, a value which is increased with greater risk aversion, reduced by increased wealth, and only slightly altered by higher limits of integration. The base measure based on cumulative prospect theory is about 46 percent less than expected damages with estimates declining when alternative parameters are used. The method of aggregation is shown to be important in the cumulative prospect case which can lead to an estimate up to 41 percent larger than expected damages. Expected damages remain a plausible and the most easily computed measure for analysts.

1. INTRODUCTION

Theoretical guidance for projects affecting risky outcomes such as flooding is complex and ambiguous. Multiple monetary measures exist based on expected utility theory as well as competing measures from other frameworks. But does the range of theoretical concerns yield an equivalently wide range of empirical measures? Or are competing theoretical measures empirically close such that other characteristics such as ease of data collection, computation and transparency become more important in the choice of a measure? A better understanding of empirical differences among measures could inform benefit-cost analyses of structural and non-structural improvements and for insurance programs, such as the National Flood Insurance Program (NFIP).

Theoretical guidance to value risks is often based on expected utility theory. Expected utility posits multiple values including willingness to pay based on expected surplus (which can be linked to expected damages), option price, and considerations such as whether complete and fair insurance markets exist [e.g. *Just, Hueth, and Schmitz*, 2005; *Graham*, 1981; *Freeman*, 1991, 1989]. A willingness to pay function linking these points generates additional possibilities depending on state (outcome) contingent payment alternatives. The option price measure is frequently deemed preferable as in some cases it meets a financing constraint and has a feasible payment mechanism, but the choice of measure remains complex within an expected utility framework [*Graham*, 1981; *Just, Hueth, and Schmitz*, 2005; *Boardman*, et al., 2011; *Cameron*, 2005] . Compounding this ambiguity, increasing concern with expected utility theory has led to theories which are not based on expected utility. Cumulative prospect theory (CPT) is a leading alternative which people weight the probability of events and assess gains or losses relative to a reference point [e.g. *Tversky and Khaneman*, 1992; *Harless and Camerer*, 1994; *Wakker*, 2010].

This paper estimates and compares alternative measures of willingness to pay to avoid flood damages by linking conceptual models and their parameterization with the HAZUS [*FEMA*, 2009] empirical model of flood damages. Data for the city of Baltimore are used for the comparisons. While many analyses focus solely on floods with an expected return period of 100 years (the 100 year flood) due to its importance in the NFIP, this analysis models a continuous set of flood return periods. While the theoretical debate is wide-ranging, policy analysis of hazards typically focus on damages conditional on the event occurring and sometimes on expected damages [e.g. *FEMA*, 2009; *Rose*, 2007; *Farrow and Shapiro*, 2009]. The default, foundation model is based on the mathematical expectation of flood damages as that measure is frequently used in applications for its ease of computation and transparency. The two classes of alternative measures are based on an expected utility model with equal payments and risk aversion—the option price; and a non-expected utility model based on the cumulative prospect theory, CPT, of Tversky and Kahneman [1991]. The analysis of flooding may also inform risk based analyses in other areas such as health, the environment, and terrorism.

The paper proceeds in Section 2 by developing the theoretical differences among expected damages, option price, and cumulative prospect theory values. Typical specifications and parameter values are also reviewed. Estimation of damages and the probability distributions for flooding based on the flood return period, R, are developed in Section 3. Section 4 presents the empirical results and sensitivity tests while Section 5 concludes.

2. ALTERNATIVE VALUE MEASURES

Expected utility models are the mainstay of the economic modeling of risk [*Eeckhoudt*, 2005; *Wakke*r, 2010]. They represent an important advance by generalizing the mathematical expectation of dollar outcomes to models of expected utility, and then assessing the monetary implications of different representations of utility. Evolution and testing of expected utility theory over decades has revealed both insights and anomalies [*Starmer*, 2000]. Both expected option price and surplus have an expected utility interpretation which is developed below while the latter can be estimated based on expected damages. A non-expected utility measure is developed as an alternative which addresses some of the behavioral anomalies observed with expected utility.

2.1 Expected utility measures

Current theory tends to favor option price, a state-independent payment, as the generally preferred willingness to pay measure for policy analysis [*Boardman, et al.*, 2011; *Cameron*, 2005]. While typically defined in a two state setting where an event occurs or it does not [e.g. *Freeman* 1989, 1992; *Graham*, 1981], option price can be defined analogously for a multi-state setting [*Cameron*, 2005]. Define:

A^*: a continuous outcome, such as floods of varying magnitudes

A : the base or reference level of flooding

$V^k(W,(A,A^*))$: an indirect utility function depending on wealth, W, flood size; and whether a payment is made, k equal 1; or not, k equal 0

$f(A^*)$: the probability density of flood size

In the absence of any payment, the no-policy expected utility is

$$\int_{A^*\,min}^{A^*\,max} V^0(W, A^*) f(A^*) dA^* \qquad (1)$$

The option price, OP, is that state independent payment for a policy which achieves the A or no flood level, and which has equal expected utility to the no-policy alternative:

$$\int_{A^*\,min}^{A^*\,max} V^1(W - OP, A) f(A^*) dA^* = \int_{A^*\,min}^{A^*\,max} V^0(W, A^*) f(A^*) dA^* \qquad (2)$$

The option price is often presented in the literature as an “ex-ante” value as it is based on the equilibration of expected utilities without being conditional on specific outcomes. Its calculation depends on the specification of the indirect utility function.

To develop the surplus concept, consider the amount a person would be willing to pay to avoid a particular level of A*, for instance the exact level of a 100 year flood. For that specific (conditional) event, a person would be willing to pay up to S(A*=100) to avoid the adverse event and achieve the same utility as the policy of doing nothing. The amount S(A*=100) is independent of the probability of the event occurring.

$$V^1(W - S(A^* = 100), A) = V^0(W, A^* = 100) \quad (3)$$

S(A*) represents a state contingent payment or willingness to pay and can be defined for all outcomes A*. Due to point by point equivalency, the expected value of Equation 3 over all outcomes is equal to the expected value of utility at the original level as in Equation 4. Hence, the surplus measure has an ex-ante interpretation as being equal to a base level of expected utility just as does option price [*Freeman*, 1991].

$$\int_{A^*\,min}^{A^*\,max} V^1(W - S(A^*), A) f(A^*) dA^* = \int_{A^*\,min}^{A^*\,max} V^0(W, A^*) f(A^*) dA^* \quad (4)$$

Although S(A*) is probability independent, the expected monetary value of that willingness to pay has been termed expected surplus and used as a welfare measure [*Freeman*, 1989; *Boardman, et al*., 2011]. The expected surplus has often been termed “ex-post” based on the probability independent equivalency in Equation 3 although it has an ex-ante interpretation as discussed above.

Historically, analysts have preferred to work with expected damages as a more directly calculable economic measure, originally assuming a person was risk neutral (indifferent between two bets of equal expected value). However, the theory of expected surplus provides an alternative interpretation for economic damages. Expected damages represent a state independent approach which, given risk aversion, represents a higher degree of utility than the state dependency associated with surplus. However, when damages are measured in a way to restore a person to an original state of utility, the expected (monetary) value of damages is equal to the expected (monetary) surplus [*Freeman*, 1989; *Boardman, et al*., 2011]. Equation 5 thus defines alternative monetary metrics for use in policy analysis in which expected damages are a monetary measure of expected surplus. Empirically, damage estimates from different flood levels will be used in this paper as the estimates of S(A*).

$$\int_{A^*\,min}^{A^*\,max} S(A^*)f(A^*)dA^* \equiv Expected\ Surplus = Expected\ Damages \qquad (5)$$

There are other payment approaches that can yield expected utility equal to the no-policy alternative [*Graham*, 1981]. For this paper, computations will be implemented by noting the common expected utility of the option price, expected surplus, and no-policy approaches as in Equation 6:

$$\int_{A^*\,min}^{A^*\,max} V^1(W-OP,A)f(A^*)dA^* = \int_{A^*\,min}^{A^*\,max} V^0(W,A^*)f(A^*)dA^* =$$

$$\int_{A^*\,min}^{A^*\,max} V^1(W-S(A^*),A)f(A^*)dA^* \qquad (6)$$

The computation of OP will result from specifying a functional form and parameters for the indirect utility function and solving the equality of the first and last integrals. The density function requires further elaboration in Section 3.

The difference between the two measures, option price and expected damages, has been shown elsewhere to depend on the difference in the marginal utility of wealth in different states of the world [*Freeman*, 1989]. For instance, if there is no insurance it may be that a dollar in a damaged state of the world is worth more than the dollar in the undamaged state. The converse is also possible. For simplification, consider two states of the world. In what may be a common assumption, the marginal utility of wealth in the "no event" state of the world, V_W^0 is assumed smaller than the marginal utility when an event occurs. In that case the option price will be larger than surplus measure. Other considerations, such as whether the amount collected by a stream of payments would be sufficient to finance a project or policy tend to favor the use of option price [*Graham*, 1981]. Existing textbook advice is that "If complete and actuarially fair insurance is unavailable against the relevant risks, then option price is the conceptually correct measure" [*Boardman, et al*., 2011, p. 211]. Consequently, the paper focuses on specifications of the utility function where option price exceeds expected surplus.

Aggregation of utility plays an important part in benefit-cost analysis whether or not risk is involved. For instance, in deterministic benefit-cost analysis, the standard aggregation assumption is that marginal utilities of income and social utility are constant across individuals [*Jones*, 2005]. Although frequently criticized, no agreed upon alternative exists. In a similar manner, the literature on utility aggregation under risk typically uses functional forms for a representative agent and homogeneous measures of risk aversion across individual even though theory demonstrates the sensitivity of a representative utility function to the distribution of wealth [*Gollier*, 2001; *Eeckhoudt, Gollier and Schlesinger*, 2005]. That standard practice is followed here by assuming functional forms for expected utility consistent with a representative aggregate agent and which are invariant with respect to the distribution of wealth [*Gollier*, 2001]. However, the CPT measure is not invariant in the same way and will provide an additional test of aggregation.

The most commonly used functional forms for utility under risk are power and certain exponential functions, each of which models consumer behavior differently. Power functions model consumer behavior for technical characteristics of risk as having constant relative risk aversion (CRRA) and declining absolute risk aversion with respect to wealth [*Gollier,* 2001; *Wakker*, 2010]. Certain exponential functions model behavior as reflecting constant absolute risk aversion (CARA) and relative risk aversion which increases in wealth. Consequently the key parameter, defined below, of the standard exponential form depends on the level of wealth for a given level of relative risk aversion.

Freeman [1989] defined three utility functions including a concave power function and two specifications of an exponential function. It is useful to replicate his specifications as there remains a lack of consensus around parameterization of expected utility models [*Gollier*, 2001; *Meyer and Meyer,* 2006] and because Freeman's context free analysis was based on a two state world which will be useful for comparison. Freeman chose parameters for the utility function as informed by the empirical literature on relative risk aversion.

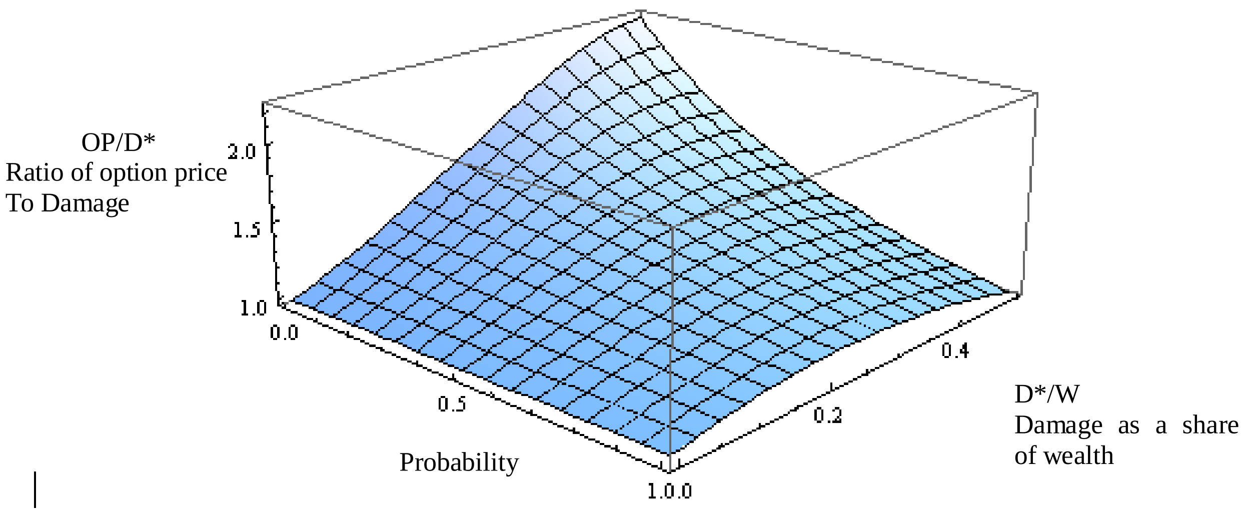
A more recent survey on measuring risk aversion by *Meyer and Meyer* [2005] investigates systematically how the definition of the outcome measure, whether wealth narrowly or broadly defined or other measures such as consumption can systematically alter empirical parameters measuring relative risk aversion. The most narrowly defined measure of wealth is based only on those assets which can be freely adjusted, as in a financial portfolio. *Meyer and Meyer* [2005, pp. 43] indicate that measured relative risk aversion in this case is generally less than 1. Assets such as housing expand the definition of wealth but may be less freely adjusted with an implication that measured relative risk is larger, perhaps in the range of 2 to 3 [*Meyer and Meyer*, 2005, p. 53]. Given the uncertainty about the role of housing in the utility of wealth, the relative risk values assumed by Freeman; .5, 2, and 10 will continue to be used in this analysis. Given these fixed relative risk aversion values, the exponential utility parameter, b, is computed based on wealth in the case to be studied. These specifications are summarized in Table 1 below.

Table 1: Specifications for the Indirect Utility Function (X is either S(A*) or OP)

Alternative	Specification	Implications	Utility Class
Power function	$V = (W\text{-}X)^{.5}$	Relative risk aversion = .5	CRRA: Constant relative risk aversion and declining absolute risk aversion
Exponential function	$V = (1\text{-}\ e^{-b(W\text{-}X)})/b$	Relative risk aversion (rr) = b*W; investigated for rr =2, 10	CARA: Constant Absolute Risk Aversion and increasing relative risk aversion

On the basis of these empirically informed specifications, Freeman concluded that it was precisely where probabilities were low but potential losses were high that the difference between option price and expected damage is large. For instance, an event leading to a loss of 50 percent of wealth with relatively small probability of .001 yielded a percentage difference between the surplus and option price values that was ten times higher than the same loss with the much higher probability of .9. Figure 2 below based on data from Freeman [1989] illustrates that the difference in the value measures in the two state case is of greater concern for events causing high damages compared to wealth and which occur very infrequently. These “tail events” are of major concern in risk management. Whether or not this large difference carries over to a multi-state case is unclear and is a further motivation for this paper. Given the potential for natural hazards to be low probability and high consequence events, it is possible that potentially large adjustments between option price and expected surplus values could significantly alter the results of standard benefit-cost analysis.

Figure 1: Ratio of Option Price to Damages



2.2 A non-expected value measure

A large and growing body of literature seeks to identify the behavioral determinants of willingness to pay after research identified numerous inconsistencies between behavior and the implications of expected utility theory [*Wakker*, 2010; *Starmer*, 2000]. Much of this research is based on the prospect theory model of Kahneman and Tversky [1979]. A key feature of this theory is that individual choices are made based on perceived probability and valuation. Prospect theory evolved further with probability weighting depending on a transformation of the cumulative distribution and hence termed cumulative prospect theory [CPT, *Tversky and Kahneman*, 1991; *Wakke*r, 2010]. One way to view this theory is as a generalization to expected utility theory where additional parameters shape the consumer response to probability and outcomes [*Machina*, 2000].

Prospect theory, and ultimately, cumulative prospect theory, built on utility theory behavior by using a probability weighting function for losses, Π(F(A*)) where F(A*) is the cumulative distribution function. Individuals are further modeled to value outcomes depending on context, c, particularly if the outcomes are favorable or unfavorable to create a value function Ṽ(W,c) that is a transformation of measured values such as damages. Such functions are generally estimated by finding points of certainty equivalence where individuals are indifferent between a risky outcome and a sure outcome, much as in Equation 4. Although originally developed for a limited number of outcomes, recent extensions develop cumulative prospect theory for continuous outcomes [*Davies and Satchell*, 2004; *Wakker*, 2010, p. 272; *Kothiyal, Vitalie and Wakker*, 2011]. Focusing only on negative outcomes for this study, a continuous representation of CPT then multiplies weighted marginal probabilities (the derivative of the probability weighting function) and the context value over all states of the world [*Davies and Satchell*, 2004; *Wakker*, 2010, p. 272]:

$$\int_{A^*\,min}^{A^*\,max} \tilde{V}(A^*, c)(\pi'_{A*}(F(A\,*)))dA^* = \int_{A^*\,min}^{A^*\,max} \tilde{V}(A^*, c)(\pi'_{F(A*)}(F(A^*))f(A^*)dA^* \qquad (7)$$

The right hand side of Equation 7 provides the more intuitive explanation. The CPT value function is weighted by the derivative (slope) of the value function with respect to its location in the cumulative density function. The expected value results when multiplied by the probability density of the event occurring, f(A*), the derivative of the cumulative distribution function. Consequently all of the measures; expected damages, option price, and CPT have interpretations as different forms of mathematical expectation.

Probability weights have been found to be affected by factors relevant to the context of flooding. For instance, perceived probabilities may depend on experience such as the "near miss" of a flood; on incorrect beliefs about the causes of an event (for instance, that levees provide perfect protection), or there may be neglect of small probabilities among other possible perceptions [*Wakker*, 2010; *Hallstrom and Smith*, 2005; *Rabin and Thaler*, 2001; *Botzen*, 2009; *Bell*, 2007, *Kahneman and Tversky*, 1979; *Tversky and Kahneman*, 1992]. Similarly, the reference point for the outcome has been shown to be central to behavioral modeling with people valuing losses differently than gains. Flooding represents losses and so may be valued differently than an equivalent amount of gains. While the issues raised in CPT are apparently relevant to flooding, estimation of the parameters of CPT are typically derived in lab settings with people making choices in the context of a financial decision. Consequently, parameters from these other contexts are used here, while noting the potential for further research for parameter estimation based specifically on the context of flooding.

Much of the CPT research uses functional forms similar to those in expected utility theory [*Wakker*, 2010]. The most frequently used is based on Tversky and Kahneman [1991] who applied a modified power function to model value with additional parameters to capture reference dependence for losses, λ; and to weight outcomes, θ. Probability weighting functions add a new modeling dimension designed to allow over or under-weighting compared to the "true" probability. Tversky and Kahneman defined a transformation of cumulative probability based on a power parameter, e. Thus a specific continuous power function representation of Equation 7 in the loss domain, pre-multiplied by the weighting function, is defined as in Equation 8 where F is the cumulative distribution function and D(A*) is the nominally measured damage outcome:

$$\int_{A^*\,min}^{A^*\,max} (-\lambda(-D(A^*)^{\theta}\ )\pi'(F(A^*))f(A^*)d\,A^* \quad (8)$$

$$where\ \pi'(F(A^*)) = \frac{d}{dF(A^*)}\ (\frac{F(A^*)^{e}}{(F(A^*)^{e}+(1-F(A^*))^{e})^{\frac{1}{e}}})$$

The storm return period, R, associated with flood modeling provides a natural ranking structure in the loss domain for A* where higher values of R rank worse. Consequently, the approach taken here for the CPT measure first investigates the combined probability and utility function for monetary losses using parameter values estimated by Tversky and Kahneman [1991; p. 311-312; *Wakke*r, 2010, p. 254-256] and then conducts sensitivity analysis. The base case parameter values were developed from experiments in which respondents chose the monetary boundary (certainty equivalent) between a certain payoff and an uncertain outcome, including uncertain losses [*Tversky and Kahneman*, 1991]. It can be shown that the loss aversion parameter, λ, is altered by the units of measurement [*Wakker*, 2010] and is adjusted for purchasing power using the consumer price index compared to the time of the experiments.

Sensitivity tests are based on research to refine probability weighting and value functions although no specific studies related to flooding have been found. Abdellaoui, Bleichrodt, and Paraschiv [2007] review a number of weighting studies with particular attention to the value function and find, in general, that the estimates are similar to those of Tversky and Kahneman although not all report a standard error. They carry out their own experiment to focus on the value function. Etchart-Vincent [2004] also reviews the literature while focusing on the probability weighting function and finds some differences in probability weighting when small and large losses are considered. Consequently, the Tversky and Kahneman parameters will be used as the base case with sensitivity based on a power parameter estimate, θ equal to .798, from Abdellaoui, Bleichrodt, and Paraschiv and a weighting function parameter, e equal to .908, for large losses reported by Etchart-Vincent for a weighting function.

Ultimately, the valuations for option price and CPT depend on utility functions whose exact form in general and for flooding in particular are unknown. However, investigating whether significant differences from expected utility using a canonical CPT function in the literature provides information about the importance of expected utility compared to a common non-expected utility model.

3. QUANTITATIVE IMPLEMENTATION

There are challenges in adapting the alternative valuation approaches to an applied setting. Implementation of Equations 5, 6 and 8 require additional data for the probability of the event, the damages, and the initial wealth. Each of these is discussed below.

3.1 Probability of flood events

The probability of a flood event is critical to estimate each of the values of interest. If the probability of a specific event exists, then the expected value calculation is straightforward. With a continuous estimate of damages, the density function of those damages is required. However, as in the case of flooding and some catastrophic analysis, the underlying analysis is based on the exceedance probability. The exceedance probability of an event such as stream flow, x , is the probability of being equal to or greater than some specific flood value, $P(x \geq x_o)$. This probability is a statement about the inverse or complementary cumulative distribution function, CCDF equal to 1-F(x) where F(x) is the usual cumulative distribution function [*Scawthorn et al*., 2006a, 2006b; *Grossi and Kunreuther*, 2005; *Chin*, 2000].

Hydrologists analyze estimated exceedance probabilities but typically describe results using the return period defined as the inverse of the exceedance probability, 1/CCDF. A statistical interpretation of this measure is the expected number of time periods, R, until a certain flood size, x_0, is exceeded [*Chin*, 2000; *Prakash*, 2004]. R is commonly called the return period. As Chin states, "it is more common to describe an event by its return period than its exceedance probability" [*Chin*, 2000, p. 257].

This common practice defines a transformation of the underlying flood random variable, x, into another random variable, R(x). The HAZUS program, to be described in the next section, uses the return period in this latter way define a given flood event, $x_{0.}$ When used in this way, the probability density function of R(x) can be derived from that of x. An informal derivation is provided here. Appendix B contains a more detailed derivation using integration by substitution.

The informal derivation asserts that the probability of exceedance in natural units, 1-F(x), should equal the same probability of exceedance when measured in terms of the return period, R(x), such that 1-F(x) is equal to 1-F(R(x)). In words, if there is a five percent chance of a flood exceeding a size x_0, then there should also be a five percent chance of a flood exceeding the transformed variable $R(x_0)$. In that case the density function of R, *f*(R), can be immediately derived by substitution and the first fundamental theorem of calculus as follows:

$1\text{-}F(x) = R(x)^{-1}$ by definition

$1\text{-}F(R) = R^{-1}$ by assumption as above and substitution, then:

$$\frac{d(1-F(R))}{dR} = \frac{dR^{-1}}{dR} \Rightarrow$$

$$f(R) = R^{-2} \qquad (9)$$

The return period R is used as the empirical measure of A* in this paper. Consequently, the density function, f(R), is used in the calculation of expected damages for each of the damage, option, and CPT measures. Further the cumulative distribution function, F(R) is used in the CPT probability weighting function as in Equation 8.

3.2 Estimation of flood damages

Forecast estimates of flooding damage are an element of each of the three measures. Floods can cause damages to structures, belongings and business inventory, affect business and personal activities and so on. Empirical estimates of such damages typically attempt to measure the cost of restoration to a pre-damaged state. Such estimates are conceptually similar to the deterministic compensating variation, the amount a person would have to be compensated in a new state of the world to be utility indifferent to the original state of the world.

The Federal Emergency Management Agency has developed a national level flood and other natural hazards damage model, HAZUS-MH [HAZUS; *FEMA*, 2009]. The HAZUS software used for this research was HAZUS-MH MR4 running with ArcGIS v. 9.3.1. HAZUS is designed to model outcomes at the census block level in its Level 1 analysis, although analysts with even more detailed information can modify the model for a higher level analysis. The model is relatively well documented and in use throughout the country [*FEMA*, 2009; *Scawthorn, et al*. 2006a, 2006b]. The damage factors included in HAZUS are dependent on the degree of flooding are building damage, contents and inventory loss, relocation, wage, and rental income loss. The largest individual components are the building and content damage [*Joyce and Scott*, 2005]. These measures do not include potential psychic effects, secondary (indirect or multiplier) effects or non-use values (for instance, if people who are never to visit New York are nonetheless harmed by learning of flood damage in New York). HAZUS computes point estimates and does not contain information about the variance of the estimate.

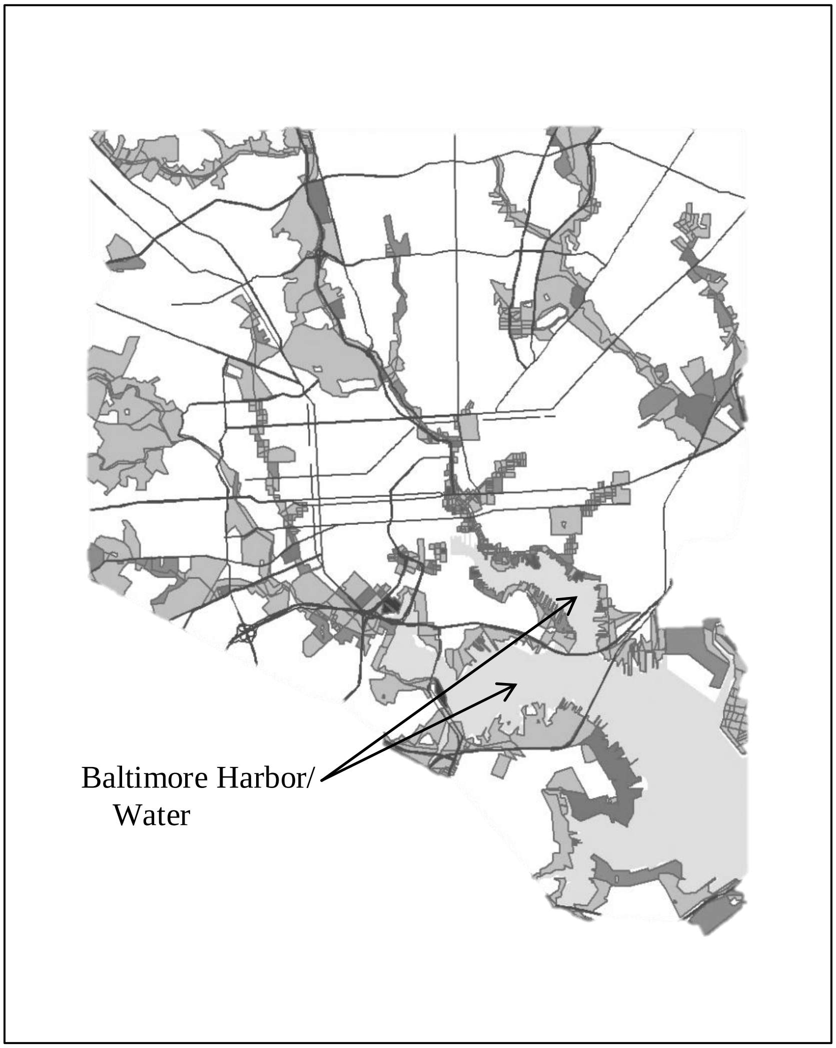
In somewhat more detail, the HAZUS flood model uses census block-level data containing information on the type and value of the building stock, employment profiles, population counts, stream gauge locations and flow volumes. Damages are estimated by linking the spatial extent and depth of a flood to the location of structures of various types and then applying historically estimated depth-damage relationships. Damage information generated by HAZUS includes counts and characteristics of buildings damaged along with monetary estimates of damages [*FEMA*, 2009; *Joyce and Scott*, 2005]. Monetary damages are based on case studies of flood events and engineering damage functions. The monetary measures of loss are: the cost of repair and replacement of buildings damaged and destroyed, the cost of damage to building contents, losses of building inventory involving contents related to business activities, relocation expense for businesses and institutions, the loss of services or sales, wage loss linked to business income loss, and rental income loss to building owners.

The exact locations of damaged buildings within a census block are not known in a Level 1 analysis. HAZUS therefore assumes buildings and associated damages are uniformly distributed throughout the census block. This assumption may be relatively reasonable in a dense urban area but less accurate in rural areas with larger census blocks. Other uncertainties arises with a Level 1 analysis. The characteristics of the building stock, such as basement occurrence or foundation height, are inferred from generalized economic census data, regional US Department of Energy data, and previous loss statistics from the NFIP. The digital elevation model used to compute stream locations, components, and drainage basins is coarser than what is potentially available. The relationship between depth of water above the first finished floor and damage to the property (the depth-damage function) is interpolated from NFIP data for several "record" floods in different regions of the country. While this level of analysis is likely appropriate for a city-wide application as in this research, researchers can improve precision through a Level 2 analysis especially if a smaller area was the focus. HAZUS provides users with the ability to import detailed flood depth studies, individual structure locations, specific foundation heights, value, mitigation factors and customized depth-damage formulas. This use of improved place-specific data can considerably reduce uncertainty and error [*FEMA*, 2009; *Scawthorn, et al*., 2006a, 2006b].

Damages are driven by the depth of flooding, which can occur due to both riverine and coastal flooding. A particular HAZUS model run is scaled by choosing a flood level defined by the return period, R. Damages are then associated with structures within and up to the boundary of a flood that is exactly that of the R year flood. The calculation of the riverine and coastal flood hazards associated with the flood size associated with any given return period are accomplished in separate processes in HAZUS. For the riverine hazard, a hydrological and hydraulics analysis is completed [*FEMA*, 2009; *Scawthorn, et al*., 2006a, 2006b]. The hydrologic analysis involves computing the expected flow volume for a return period using regional regression equations to predict stream discharge amounts and drainage basin size. The hydraulic analysis then interpolates the flood elevations and the floodway based on the expected flow volume and the stream channel characteristics. The user selects the spatial level of detail which determines how many stream reaches or tributaries will be included in the hydrological analysis with correspondingly increased computational requirements for additional reaches. For coastal flooding, the shoreline must be characterized by both the degree of wave exposure (from sheltered to full exposure) and the shoreline morphology (such as rocky or large dunes). When both coastal and riverine flooding occurs in the same area, the model picks the "predominant" flooding mechanism and its associated flood depth.

The HAZUS model output data were used to estimate a function, D(R), linking the flood return period to the level of damages for the City of Baltimore, a defined region within the HAZUS model. This is the empirical basis for damages in the several valuation measures. Baltimore City is subject to both riverine and coastal flooding. For the HAZUS runs, the computable number of riverine reaches was between 60 and 80 implying a modeled drainage area for each reach of about 1 square mile**.** This computable number of reaches depends on both the HAZUS version and the computing resources available. One full run of the model required about a day of computer run time. Figure 2 displays the estimated damages for a return period equal to 100, the size flood with a 1 percent annual chance of occurring or being exceeded. The total estimated damage from a 100 year flood in Baltimore City is $837 million in 2008 dollars. Structural damages are $272 million of that total. The value of total building exposure in Baltimore City within the 100 year floodplain is approximately $1 billion [*Joyce and Scott*, 2005].

Figure 2: Damages from the 100 Year Flood: Baltimore City (darker area higher damages)



In order to estimate damages as a function of the return period, the HAZUS model was run for nine different return periods; 10, 30, 50, 75, 100, 150, 200, 350, and 500 years. Regression analysis was used to generate a line of best fit to the data. The results for a logarithmic regression of damages on the flood period are presented in Table 2. The return period is highly significant and the measure of fit is high. Diminishing marginal damages exist as the elasticity of damages with respect to return period is .25; a one percent increase in the return period lead to a .25 percent increase in damages.

Table 2: Estimated equation for damages: Baltimore City

Dependent Var. Ln Total Damage in 000 dollars	**Coefficient**	**Standard Error**	**t Value**
Constant	12.3899	.0672	184.48
Ln Flood Return Period (R)	.2537	.014	17.72
Observations 9, Adj. R^2 = .97, Root MSE= .04976			

Estimated losses from a one year storm, R equal to 1, are constrained to be zero for each of the three value measures. Hence the one year return period is the reference point for the CPT measure and damages are estimated as those losses that exceed those for the base flood, a flood that is expected to be exceeded every year.

The 100 year flood, R equal to 100, is an important policy benchmark due to the NFIP. That program requires insurance for owners within the 100 year flood plain who have a federally backed mortgage or who obtained a mortgage from a regulated lender [*Kousky, Luttmer, and Zeckhauser*, 2006]. The insurance contains standard provisions such as a deductible, a cap, and limitations on the type of damages covered. Total damages estimated by HAZUS are not the same as potential insured losses under the NFIP due to partial take-up rate on NFIP insurance and the limiting provisions. In the period from 1978 to 2010, the highest NFIP claims paid were in 2003 in the amount of $6.8 million. In ten of the 33 years, no claims were paid [*Howard*, 2012]. However, broader programmatic analyses and reviews of the NFIP are likely to be concerned about damages from the entire distribution of potential floods, damages not covered by insurance; and uncertainty about valuation measures such as the option price, expected damages, and CPT measures as developed above.

It is also useful to note the case specific role of the damage function. Here the estimate is of increasing but diminishing marginal damages. In other contexts such as homeland security or perhaps for the largest floods, the damage function may increase at an increasing rate up to some point as systematic linkages among damaged parts of the area could change the shape of the damage function.

3.3 Wealth

The definition of the wealth or income over which the individual is averse can significantly affect results [*Meyer and Meyer*, 2006]. For instance, Freeman [1989] developed his approach using income although wealth seems the more appropriate asset in this case. Freeman's maximum damage as a share of income was 50 percent. For major events such as floods or terrorism, some individuals may well suffer losses significantly exceeding 50 percent of wealth although some specific forms of utility functions become undefined if the loss exceeds total wealth. The measure of wealth used here is based on the value of improvements in the 100 year flood plain, $1 billion, as approximately adjusted for the extent of larger floods and other elements of total damages included in HAZUS such as contents and inventory loss. The resulting value used for the base case for Baltimore City is $5 billion dollars and $97 billion as a sensitivity analysis for the total city exposure [*FEMA*, 2009; *Joyce and Scott*, 2005].

3.4 Estimation

Two key steps are common for each of the three measures: expected damage, option price, and CPT value. Those steps are the estimation of individual components at each (continuous) flood level and the computation of expected value via numerical integration. In addition, the computation of option price requires solving two integral equations for a value that makes them equal, the option price. The final computation of each measure is summarized as below.

The expected damage estimate of Equation 5 is computed using the density function of R, f(R) for f(A*) from Equation 8 and the damage function D(R) for S(A*) from Table 2 measured as a difference from the one year flood estimate. While the expected value integral admits of a closed form solution, the results are obtained numerically using Mathematica8 [*Wolfram*, 2011] as later measures require numerical computation. The limits of integration are taken to be 1 and 500 where the lower bound is the level of flood that is expected to be exceeded every year and the upper bound is a flood which is expected to be exceeded every 500 years (although each annual outcome is independent). The impact of the upper limit is investigated through sensitivity analysis.

Three different measures of option price are computed based on the differing utility specifications in Table 1. For each specification, the option price is calculated from Equation 2 noting the utility equivalency in Equation 6 to the utility of expected surplus. The density and damage functions f(R) and D(R) are used as above along with wealth from section 3.4. Integration and the solution to Equation 2 is found using Mathematica8 [*Wolfram*, 2011]. The solutions were checked by determining that Equation 2 holds. As suggested by *Wakker* [2010], the exponential form of the utility function in Table 2 is preferred to the economically equivalent form presented in Freeman [1989].

The estimation of the CPT value is computationally similar to that of expected damages although the cumulative probability, F(R), and damage, D(R), functions are shifted by three parameters. Those parameters are λ, θ, and e as defined in Equation 8 with values described in section 2.2. The expected CPT value of Equation 8 is obtained numerically using Mathematica8 [*Wolfram*, 2011].

4. RESULTS

The quantitative results of the two expected utility measures, expected damage and option price measures, and the CPT value are reported in Table 3. Parametric sensitivity results are also reported in Table 3 and others are discussed in the text.

Total expected annual damages for riverine and coastal flooding in Baltimore City is $79 million as reported in row 1. Recall that the damage estimate includes damage to buildings as well as other elements of business damage. Building damage is about one-third of the total. Although computed as expected damages, the measure also has an interpretation as equal to the monetary value of expected surplus when damages are comprehensively measured. This measure represents the base case against which other measures will be compared.

Rows 2 through 5 are all option price measures. Each numbered row has results for the three different utility specifications which are identified by the parameter for relative risk aversion in column 2.

The basic option price results are presented in row 2. For measures of relative risk aversion most representative of the literature, .5 for the power function and 2 for the exponential form, the option price result is quite close to expected damages, $80 and $81 million respectively. The basic option price adjustment to expected damages leads to increases of only a few percent as reported in the last column. If the utility function exhibited high risk aversion with relative risk aversion equal to 10, then the option price is estimated as $92 million; sixteen percent higher than expected damages.

One might have anticipated from Figure 1 that the use of option price would lead to a large increase over expected damages as, for instance, the 500 year flood damage represents almost a 20 percent loss of wealth. However, the probability of such a large flood is small so that the (expected) option price represents only a modest increase over expected damages given the conditions of this case.

Table 3: Expected value results and sensitivity testing

Scenario			**Upper / lower limit**	**WTP Total Mil.2008 $**	**% Change from E(D)**
EU Measures	**Relative Risk Aversion**	**Wealth**			
1. Expected Damage E(D) (Surplus)	--	5 B	500/1 100/1	**$ 79** **$ 74**	**0%** **0%**
2. Option Price	.5 2 10	5 B	500/1	**$ 80** **81** **92**	**1%** **3%** **16%**
3.Option price -- Upper limit of integration	.5 2 10	5 B	1000/1	**$ 81** **82** **94**	**3%** **4%** **19%**
4. Option Price - High Wealth	.5 2 10	97 B	500/1	**$ 79** **79** **80**	**0%** **0%** **1%**

Non-EU Measures	**Power Value(s)**	**Probability Weight**	**Upper /lower limit**	**WTP Total Mil. $2008 Rep.**	**Avg.**	**% Change from E(D) Rep.**	**Avg.**
6. Base CPT	.88 (λ=2.25) 88 (λ=2.25)	.69 .69	500/1 100/1	**$ 43** **$35**	**$ 111** **$90**	**-46%** **-53%**	**41%** **22%**
7. Base CPT lower limit =2	.88 (λ=2.25)	.69	500/2	**$ 40**	**$ 102**	**-49%**	**29%**
8. CPT estimate prob	.88 (λ=2.25)	None	500/1	**$ 31**	**$80**	**-61%**	**1%**
9. CPT w/Alt prob weight	.88 (λ=2.25)	.908	500/1	**$ 33**	**$ 85**	**-58%**	**8%**
10. CPT w/alt value coeff.	**.**798 (λ=2.04)	.69	500/1	**$ 8**	**$ 40**	**-90%**	**-49%**

Source: Author's calculations

Sensitivity tests of the option price model are presented in rows 3 through 5. Row 3 doubles the upper limit of integration to 1,000; twice the base upper limit and well beyond the data on which the damage equation is estimated. The increase in the upper limit increases option price to $81 million, about a one percent increase over the option price estimate based a 500 year limit of integration and three percent larger than expected damages. This sensitivity test reinforces the hypothesis that the expected value calculation is reducing the effect of very low probability but high damage events. A second sensitivity test in row 4 increases the exposed wealth to the improved value of all of Baltimore City. The larger wealth reduces the premium that people would be willing to pay such that the option value is equivalent to expected damages for two of the specifications and only slightly increases the option price to $80 million in the highly risk averse specification. Additionally, in the case of relative risk equal to .5, where the expected utility specifications can be compared without violating parameter conditions. The difference in the results was minimal, less than $1 million (results not in table).

Consequently the first conclusion is that the option price measure of willingness to pay is only a small adjustment to expected damages unless there is a very high level of risk aversion in which case there is less than a 20 percent difference between the expected utility measures.

Estimates based on cumulative prospect theory begin in row 6 (expressed as positive willingness to pay). The base CPT estimate using parameter values from Tversky and Kahneman [1992] is $ 43 million, about 46 percent less than the expected damage estimate. The CPT value function plays an important role in understanding why the CPT estimate is less than the expected damage and option price measures. Given the parameter values, the damage measure exceeds the CPT value for most of the range of integration.

An intended aspect of CPT is that the weighting function over-weights events with both small and large outcomes, and under-weights in between. This effect can be seen in several ways. Plots of the data, not shown here, indicate the base weighting function over-weights flooding compared to the unweighted probability between return periods of 1 and about 1.1 and slightly over-weights floods with return periods greater than 6. The monetized effect can be seen in several sensitivity tests. In row 8, there is no weighting of the value measure, only the density function of the return period is used to construct the expected value. This decreases the expected value to $31 million indicating that overall the weighting function serves to increase the CPT measure compared to an unweighted value function. Additionally, in row 7, the limit of integration ends at a return period of 2. The resulting expected value is $ 40 million, a 7 percent decline from the larger range of integration indicating a moderate amount of the CPT value lies in very small floods below a return period of 2. Secondly, if the limit of integration is increased to infinity (far beyond any estimation of the damage function), the value increases to $52 million, a 21 percent increase over the base rate (not reported in Table 3). As 99.8 percent of the probability of flooding is between return periods of 1 and 500, the remaining 1.2 percent of possible outcomes does have a discernible but not dramatic effect on the outcome.

Variations of the CPT parameters only serve to reduce the estimate for willingness to pay. An alternative probability weighting function from Etchart-Vincent [2004] in row 9 leads to a 23 percent reduction from the base CPT case to $ 33 million (a 58 percent decrease from expected damages). The alternative value function parameters from Abdellaoui, Bleichrodt, and Paraschiv [2007] are used in row 10 but the base model probability weighting function is maintained. These parameters yield a significantly lower value than the base case, $ 8 million, indicating that alternative parameterization of the value function can also have a significant impact.

Consequently, the second conclusion is that the CPT measure applied to the aggregate is uniformly less than the expected damage and option price values. Sensitivity tests of the parameters tended to reinforce the lower estimate of willingness to pay.

However, the loss aversion incorporated into CPT measure such that smaller losses have larger relative weight than larger losses can be shown to have a significant effect. This returns to the issue of the representative agent in aggregating values. In the case of the CPT measure, computing an average value of damages, and then aggregating it across those damaged, can lead to a significant increase over and above the expected damage or option price measures. Results for the CPT measure based on computing the average value per building damaged in a 100 year flood, and then aggregating by the number of owners are presented next to the representative agent results for the CPT value. The average CPT value using the base Tversky and Kahneman parameters are 41 and 29 percent above the expected damage estimate as reported in rows 6 and 7. The parametric sensitivity tests in rows 9 and 10 reduce the estimate first to a level more representative of the expected damage and option price values, $80 million; and then to a value smaller than those estimates, $40 million.

Consequently, disaggregation is important in the CPT measure in a way that is not apparent with specific but standard forms of the expected utility function.

The NFIP is focused on providing insurance to those within the 100 year flood plain. In order to assess the overlap between the focus of the NFIP and total damages, the expected damage and CPT value were recomputed based only on return periods between 1 and 100. The result, in rows 1 and 6, demonstrates that that most of the willingness to pay exists within the 100 year return period. The expected damage measure falls from $79 to $74 million when all the floods in excess of the 100 year flood are ignored. Similarly, the CPT value measure declines from $43 million to $35 million when the same larger floods are ignored. This is further indication that the expected value measures change relatively little from the larger and more damaging but less frequent floods beyond the 100 year flood.

5. DISCUSSION AND CONCLUSION

The empirical results reported here differ from the casual implications of Figure 1 and some ad-hoc expectations with respect to a behavioral model. The results of the three measures; expected damages, option price, and CPT value and their sensitivities indicate for flooding in Baltimore City that:

1. There is minimal difference between the expected damage and the option value measures of willingness to pay when standard levels of risk aversion are used.
2. The difference between expected damages and option price can become larger if a sufficiently large degree of risk aversion exists but the difference is less than 20 percent.
3. The results for option price are little changed when either the upper limit of integration is increased or the magnitude of wealth is increased.
4. The representative agent CPT estimates, a non-expected utility framing, are significantly less than either of the expected utility models.
5. Variations on the representative agent CPT parameters further reduce the CPT measure.
6. Disaggregating the CPT measure can but need not reverse the conclusion. The average CPT value with the base parameters is larger than expected damages or option price although alternative parameterizations can reduce the average CPT measure below expected damages.
7. Expected damages and the base CPT values are only moderately changed when the limits of integration focus on the limits of concern to the NFIP, the damages due to a 100 year flood or less.

The case study here has important assumptions which are worth reviewing and which indicate directions for further research. The case study is built on a multi-state, continuous outcome setting which may correspond to many natural and man-made hazards. The case specific damage function is increasing at a decreasing rate which may not be representative of all cases and also affect convergence and solutions. Statistical uncertainty is not yet a component of the damage estimates from HAZUS. This absence of statistical uncertainty about the expected values would likely reinforce the closeness of the measures as reported above. The specification and parameters of the functions, while informed by the literature, are not specific to the case of flooding and have the strengths and weaknesses of laboratory based estimates. Concerns about systematic risk in a region if wide-spread damage occurs is not included which may lead to larger than estimated damages for very large events.

With the above cautions however, it appears that this case identifies two important modeling choices for analysts. The first choice is the use of an expected utility or a non-expected utility analysis. Expected damages and option price appear to provide similar results for the parameters and case studied while the CPT value is significantly less. Secondly, aggregation is demonstrated to have an important effect for the CPT value and may have important effects if more flexible forms are used for the expected utility analysis. The encouraging result for analysts faced with multiple, complex measures for computation is that expected damage does not appear to be an outlier and could remain the standard default measure unless further investigation reveals otherwise.

Finally, extremely large floods have relatively little effect on expected value measures. This is demonstrated both by small changes, for expected utility measures, and moderate changes for the CPT value when the upper limit of integration is increased. Further, when the limits of integration reflects the focus of the NFIP program being less than or equal to the 100 year flood, then a large part of the expected value measures is captured within that limit. While not inconsistent with current policy, the result also suggests the usefulness of research on different objective functions than expected value.

Appendix A: Alternative derivation of the density function for R(x)

Define

x: flood measure (height or flow, a non-negative value);

F(x) cumulative distribution function of x with density function f(x)

R(x) ≡ 1/(1-F(x)) which is a monotonic transformation of x given the properties of F(x). Since F(x) is increasing in x, R(x) is increasing in x.

Apply integration by substitution to R(x). Then

$$\int_{xmin}^{xmax} f(R)\frac{dR}{dx}dx = \int_{R(xmin)}^{R(xmax)} f(R)dR$$

Substituting dR/dx equal to R^2 f(x) from above, then

$$\int_{xmin}^{xmax} f(R)R^2 f(x)dx = \int_{Rmin}^{Rmax} f(R)dR$$

Consequently,

$$f(R)R^2F(x) = F(R)$$

$$f(R) = \frac{F(R)}{F(x)}R^{-2}$$

The density function of R is then seen to be equal to R^{-2} if the cumulative distribution function F(R) equals F(x) which is asserted to be the intent of the transformation. This derivation provides a further clarification of the role of equivalent cumulative distribution functions which was used in the more intuitive derivation in the text.

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