The Importance of Production Networks and Sectoral Heterogeneity for Monetary Policy*

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Abstract

In this paper I develop a multi-sector model with price frictions, production networks, trend inflation and different types of shocks to study how these conditions affect the properties of inflation and their implications for monetary policy. Calibrating the model to the U.S. economy my results show that in this setting inflation becomes 30% less sensitive to the output-gap compared to a standard one-sector model. Furthermore, in the multi-sector model inflation is affected by sectoral variables linked to between-sector and within-sector price distortions. This fact adds inertia to the inflationary process and makes monetary policy less effective. Additionally, the welfare costs of trend inflation increase by one order of magnitude in the multi-sector model compared to the standard one-sector model. The amplification is quantitatively explained by between-sector rather than within-sector price distortions. This suggests that one-sector models and models without heterogeneity underestimate the costs of long-run inflation and the efficacy of monetary policy to fight inflation.

Keywords: production networks, sectoral heterogeneity, trend inflation, monetary policy

JEL classification: E30, E31, E32, E52, E58.

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1 Introduction

Most models in macroeconomics ignore the complexity of the real productive structure by assuming the existence of a single firm or a single productive sector instead of an interconnected network of firms and industries.[1] This one-sector assumption has deeply permeated the macroeconomic analysis even under circumstances where the input-output linkages between firms or industries may have first-order consequences in aggregate variables (e.g. Acemoglu et al. 2012; Baqaee and Farhi 2017).

In this paper I depart from the one-sector assumption and study an economy with monetary frictions where industries are connected through input-output linkages, and where there exists sectoral heterogeneity across these industries. By studying the properties of this economy I find relevant implications for monetary policy. Specifically, compared to the standard one-sector model, aggregate inflation reacts 30% less to the output-gap, rendering monetary policy less effective, and the welfare cost of trend inflation increases by one order of magnitude.

To reach these conclusions I develop a multi-sector model with production networks, aggregate and idiosyncratic productivity shocks. The model features nominal frictions as in Calvo (1983) but allows for sectors to differ in their frequencies of price change. The monetary authority sets an exogenously chosen rule to determine the nominal interest rate and a target of trend inflation, i.e. the level of inflation observed in the long run. I calibrate this model for the U.S. economy and compare the results with its one-sector version.

The calibrated multi-sector model presents some key distinctions from the one-sector economy one in terms of inflation dynamics and welfare losses associated with trend inflation. First, in terms of inflation dynamics, aggregate inflation in the multi-sector economy is 30% less sensitive to the output-gap, measured as the coefficient of the New-Keynesian Philips curve (NKPC). Sectoral heterogeneity —in the form of sectoral differences in the frequencies of price change and/or sectoral productivity shocks– affects inflation through within-industry and between-industry price distortions. This fact also reduces the relevance of aggregate output on affecting inflation, making monetary policy less effective and giving inflation higher inertia. Furthermore, higher trend inflation increases the likelihood of finding multiple equilibria in the multi-sector model, thus dynamic responses to shocks become more difficult to predict.

Second, in terms of the welfare losses from trend inflation, the multi-sector model yields larger

[1] Henceforth I use the terms sector and industry interchangeably.

consumption-equivalent welfare losses than the one-sector model even at moderate levels of inflation. Given the calibration of the model, where trend inflation is 2% annually, the welfare loss in the multi-sector model is 0.34% compared to 0.03% in the one-sector economy. Trend inflation affects welfare by decreasing labor productivity and by rising the volatility of consumption and labor. Productivity losses diminish welfare by increasing the amount of labor needed to produce any given level of output. Higher volatility of consumption and labor reduce welfare when individuals are risk averse. These effects are present in both the one-sector and the multi-sector economy, but are more important in the latter.

There are three main mechanisms explaining these results. First, as discussed in the monetary literature, price frictions induce inefficient price dispersion across firms since firms cannot constantly change their desired relative price given the shocks of the economy. I call these frictions *within-sector price distortions* since in a multi-sector context with atomistic firms, each firm tries to set a target for its price relative to their own sectoral price index. These relative price inefficiencies translate into output inefficiencies and, in a one-sector model, are equivalent to a lower level of aggregate productivity (e.g. Yun, 2005). Higher inflation targets increase this measure of price distortion, compromising aggregate productivity even more (e.g. Ascari and Sbordone 2014). In the multi-sector model these forces are present at the sectoral level, creating distortions in the optimal bundle of goods.

The second mechanism explaining my results is what I call *between-sector price distortions,* which is a measure of sectoral relative price deviations with respect to their optimal level. In a world without price frictions, all firms within an industry can instantaneously adjust their prices when facing external shocks. In this context, a sectoral price index would always reflect these optimal price changes. However, this is not the case in a setting with price frictions. These frictions add sluggishness to prices, since not all firms can adjust prices simultaneously. When industries adjust prices at different frequencies, or when they adjust at the same frequency but face idiosyncratic shocks, sectoral price indices take longer to reflect their optimal relative values.[2] Therefore between-sector price distortions appear.

The final mechanism explaining the results is the presence of production networks. Within-sector price distortions create inefficient allocations between the goods of firms within the same sector, and between-sector price distortions negatively affect the optimal allocation between bun-

[2]See Carvalho (2006), Nakamura and Steinsson (2010) and Pasten et al. (2016) for discussions of the effects of heterogeneity in the frequencies of price change on monetary non-neutrality.

dles of sectoral goods. When goods are also used as intermediate inputs in other industries, these inefficiencies are transmitted through the productive process. As a result, aggregating the economy with production networks amplifies the inefficient allocations, creating higher aggregate productivity losses compared to a one-sector model.[3] The results of this paper can be explained in light of these three mechanisms.

The sensitivity of inflation to the output-gap. In the standard one-sector model, the slope of the NKPC is inversely related to the degree of strategic complementarity of price setting (Woodford, 2011). When pricing decisions are strategic complements, a firm's optimal price increases as other firms increase their own prices. When prices are sticky, strategic complementarity slows optimal price adjustments to shocks. Even if a firm has the opportunity to adjust, its optimal price remains relatively unchanged if other firms have not had the opportunity to adjust yet. As a result, strategic complementarity slows down the adjustment rate of the aggregate price index. This maps into a flatter NKPC since inflation becomes less sensitive to the shocks affecting aggregate output. Additionally, the presence of production networks and, therefore of intermediate inputs, increases the degree of strategic complementarity of price setting (Basu, 1995).

To measure the sensitivity of inflation to the output-gap in the multi-sector economy I obtain the NKPC of the model, which shows the relationship between inflation and the output-gap. I call this the generalized NKPC since it embeds different versions of this equation as special cases.[4] Under the baseline calibration of the model, the coefficient of the output-gap is 30% lower compared to the one-sector counterpart. Equally important is the fact that the generalized NKPC includes variables linked to the between-sector and within-sector price distortions. Furthermore, the dynamic behavior of these variables is backward looking, which adds inertia to the inflationary process and better fits the data. On the other hand, the aggregate relevance of a sector's price distortions depends on different sector characteristics. Both within-sector and between-sector price distortions have a higher impact on aggregate inflation when the sector has higher frequency of price changes and when the sector is an important supplier of inputs into the economy.

[3] In Castro Cienfuegos and Loria (2017) we study this effect by focusing on the determinants of aggregate TFP in a setting similar to the one described in this paper.

[4] The generalized NKPC embeds the NKPC of Ascari (2004), Carvalho (2006), Pasten et al. (2016) and the standard NKPC as special cases. See Appendix C for details.

Welfare losses of trend inflation. The amplification of welfare losses due to trend inflation is also explained by the three mechanisms detailed above. In a one-sector model, higher trend inflation increases inefficient price dispersion across firms, since relative prices drift away faster from their optimal levels.[5] Giving the opportunity of changing its price, a firm would do so by a larger amount, increasing inefficient price dispersion and lowering productivity. This is the main cause of steady-state output losses due to trend inflation in one-sector models like the one in Ascari (2004). In a multi-sector model, trend inflation makes every sector less efficient because of this mechanism, and production networks amplify this loss.

Higher trend inflation also increases the rate at which inefficient relative prices drift away, increasing the distortions associated with changes in relative sectoral prices and leading to inefficiencies in consumption and production. My results show that the loss of efficiency due to distortions in sectoral prices is the leading quantitative cause of output losses due to trend inflation. This result is new and relevant for discussions related to optimal inflation targets since it highlights the importance of using multi-sector models to correctly measure inflation costs. In a recent paper Nakamura et al. (2018) have argued that there is no empirical evidence of a positive relationship between price dispersion and inflation, implying that the costs of inflation are overstated in the standard New-Keynesian (NK) model relative to the menu cost model. However, my results suggest that in a multi-sector NK model, the costs of higher inflation are linked to between-sector rather than within-sector price distortions and point to the importance of taking multi-sector models into account when assessing inflation costs.

Outline. Section 2 of this paper discusses the related literature. Section 3 presents the setting of the model, and Section 4 highlights some important features of the equilibrium with price distortions and production networks. Section 5 describes the calibration of the model. Section 6 studies the dynamic properties of aggregate inflation comparing the multi-sector model with its one-sector counterpart. Section 7 quantifies the welfare costs of trend inflation and discusses some implications for monetary policy. Finally, Section 8 concludes the paper, and Section 9 presents tables and figures.

[5] For a formal proof of the relationship between inflation and price dispersion please refer to Woodford (2011), chapter 6.

2 Related literature

This paper is related to a branch of the economic literature that studies the relevance of multi-sector models for aggregate outcomes. Different papers in this literature have focused on the importance of sectoral shocks to explain aggregate fluctuations (Horvath, 1998, 2000; Acemoglu et al., 2012; Carvalho, 2014; Tahbaz-Salehi et al., 2017; Atalay, 2017; Pasten et al., 2018). Most of these papers have found, either empirically or theoretically, that idiosyncratic shocks can be largely amplified and explain fluctuations of aggregate variables in an economy with production networks. I build on this previous work by taking the production network with sectoral shocks framework into a context with monetary frictions.

It also addresses a branch of the economic literature that studies the degree of monetary non-neutrality in multi-sector models or models with different types of heterogeneity. Basu (1995) shows that the input-output structure can amplify the degree of monetary non-neutrality by increasing the strategic complementarity of price setting. Carvalho (2006) shows that the degree of strategic complementarity increases in a time-dependent model with heterogeneous frequencies of price setting, increasing monetary non-neutrality. Nakamura and Steinsson (2010) develop a multi-sector model with menu costs in a setting with production networks and find that input-output linkages along with differences in the frequency of price changes can amplify the degree of monetary non-neutrality by a factor of 9. Pasten et al. (2016) study the degree of monetary non-neutrality in a multi-sector setting with price stickiness as in Calvo (1983), exploring the quantitative importance of different types of heterogeneities by calibrating the economy for 350 sectors. They find that – although input-output matrices amplify the real effects of monetary shocks – heterogeneity of these linkages plays only a minor role versus other types of heterogeneity. In particular the real effects of monetary shocks rise when the share of intermediate inputs is more important and under heterogeneity in the frequency of price changes. My paper is related to these models particularly the final one, but my model includes sectoral productivity shocks and sectoral trend inflation. Moreover, I do not focus on studying the degree of monetary non-neutrality. Instead, I focus on how heterogeneity affects aggregate inflation, and the implications in terms of monetary policy.

My paper also speaks to academic and policy discussions about the optimal inflation target. The costs of inflation through increases in price dispersion have been stressed by some authors (Yun, 2005; Ascari, 2004; Ascari and Sbordone, 2014). However in recent years researchers and policy makers have argued in favor of higher inflation targets as a precautionary measure to avoid the costs associated with the zero lower bound constraint on nominal interest rates (Dell'Ariccia et al., 2010; Ball, 2014; Blanco, 2016). In this context, a proper balance of the costs and benefits of higher inflation targets is necessary. In a recent paper Nakamura et al. (2018) argue that there is no empirical evidence of the inflation costs predicted by the standard NK model. My results suggest that in a multi-sector model with heterogeneity there are additional sources of welfare costs (between-sector distortions), different from the ones predicted in the one-sector NK model. Furthermore, these distortions are relevant in creating welfare losses even at moderate inflation levels. Although it is out of the scope of this paper this effect could also be present in multi-sector menu-costs models, which points to the necessity of adopting multi-sectors models to properly measure the costs of inflation.

3 Setting

The economy is composed by N different productive sectors denoted by $i = 1, ..., N$, each one consisting of a continuum of firms indexed by $j \in [0, 1]$, producing a differentiated good that can be used in consumption and/or production. Within each sector firms face monopolistic competition, produce with the same production function, and face the same sectoral-specific productivity shocks. As in Calvo (1983), I assume firms can re-optimize their prices with an exogenous probability $1-\theta_i$, where θ_i is the parameter of price stickiness of sector i.

One of the main differences between this model and the standard NK one is that, besides labor, a given firm uses as inputs goods produced by firms in (possibly) every other sector of the economy.

On the consumption side, I assume the economy is represented by a single household that chooses an aggregate consumption index composed by sectoral consumption bundles. At the sectoral level, consumption bundles are constant elasticity of substitution (CES) aggregators of the goods produced by individual firms within each sector so that individual firms face downward sloping demands. The household also chooses aggregate labor and savings. The following subsections give details about the optimization problems faced by each agent in this environment.

3.1 Household

The representative household maximizes the the inter-temporal utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on information available at time $t = 0$, C_t is an aggregate consumption bundle, L_t is an aggregate labor bundle, β is the subjective discount factor, $\sigma \geq 1$ is the inter-temporal elasticity of substitution, and $\varphi \geq 0$ is the inverse of the Frisch elasticity of labor. The aggregate labor bundle is a composite of labor supplied at the sectoral level, $L_{i,t}$,

$$L_t \ := \ \left(\sum_{i=1}^{N} L_{i,t}^{1+\varphi} \right)^{\frac{1}{1+\varphi}}.$$

On the other hand, the aggregate consumption bundle C_t is a Cobb-Douglas aggregator of sectoral consumption bundles, i.e.

$$C_t \ := \ \prod_{i=1}^{N} C_{i,t}^{\alpha_i},$$

where α_i is the share of sectoral consumption from sector i and $\sum_{i=1}^{N} \alpha_i = 1$. Since each sector is composed by individual firms indexed in $[0, 1]$, the sectoral consumption bundles $C_{i,t}$ are themselves composite consumption goods defined by the CES aggregators:

$$C_{i,t} := \left(\int_0^1 C_{(i,j),t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $C_{(i,j),t}$ is the quantity of a good sold by a firm j in industry i at time t, and ϵ is the constant elasticity of substitution assumed to be the same across industries. The budget constraint of the representative household is given by:

$$P_t C_t + Q_t B_t \;=\; B_{t-1} + L_t W_t + D_t \tag{2}$$

where P_t is the aggregate price index defined as the minimum cost of consuming one unit of the aggregate composite good C_t, B_t denotes holdings of one-period discount bonds, Q_t is the nominal price of bonds, and W_t is an aggregate index of sectoral wage rates $W_{i,t}$ defined by:

$$W_t : \;=\; \left(\sum_{i=1}^{N} W_{i,t}^{\frac{1+\varphi}{\varphi}} \right)^{\frac{\varphi}{1+\varphi}},$$

and where $D_t = \sum_{i=1}^{N} \int_0^1 D_{(i,j),t} dj$ are the aggregate dividends received from ownership of all firms in the economy.

The problem for the household is thus to choose consumption C_t, labor L_t and savings B_t to maximize the lifetime utility (1) subject to the aggregate budget constraint (2). From the optimality conditions of this problem we get:

$$\frac{L_t^{\varphi}}{C_t^{-\sigma}} \;=\; \frac{W_t}{P_t} \tag{3}$$

$$1 \;=\; \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{I_t}{\Pi_{t+1}} \right] \tag{4}$$

where $\Pi_{t+1} := \frac{P_{t+1}}{P_t}$ is the aggregate gross inflation rate between period t and $t+1$ and $I_t = Q_t^{-1}$ is the gross nominal interest rate.

Given a solution to the problem to the household's problem, cost minimization of the aggregate consumption bundle yields the sectoral demand function:

$$C_{i,t} \;=\; \alpha_i C_t \frac{P_t}{P_{i,t}}, \tag{5}$$

where the aggregate price index is defined as $P_t := \prod_{i=1}^{N} \left(\frac{P_{i,t}}{\alpha_i} \right)^{\alpha_i}$. Analogously, given a solution for sectoral consumption bundles $C_{i,t}$, cost minimization produces the demand for goods of individual firms j in each sector i,

$$C_{(i,j),t} = \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} C_{i,t}, \tag{6}$$

where $P_{i,t} := \left(\int_0^1 P_{(i,j),t}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ is industry i's sectoral price index and where $P_{(i,j),t}$ is the price charged by an individual firm j in sector i at time t.

Finally, given aggregate labor supplied L_t, optimal allocation of sectoral labor – holding labor income constant – yields the sectoral labor supply

$$L_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{\frac{1}{\varphi}} L_t. \tag{7}$$

3.2 Monetary Policy

Both the nominal interest rate and trend inflation, I_t and $\overline{\Pi}$, are also exogenously determined. The nominal interest rate is determined by an exogenous Taylor rule, which sets this rate depending on GDP (which in this model is equal to aggregate consumption C_t) and aggregate inflation $\Pi_t = \prod_{i=1}^{N} \Pi_{i,t}^{\alpha_i}$,

$$\frac{I_t}{\overline{I}} = \left(\frac{I_{t-1}}{\overline{I}}\right)^{\rho_I} \left[\left(\frac{C_t}{\overline{C}}\right)^{\phi_c} \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\right]^{1-\rho_I} Z_t^m. \tag{8}$$

The variables with overbar represent values in the steady state equilibrium, and Z_t^m is a monetary shock with logarithm $z_t^m := \log Z_t^m$ following the AR(1) process $z_t^m = \rho_m z_{t-1}^m + \varepsilon_t^m$ and $\varepsilon_t^m \sim N\left(0, \sigma_m^2\right)$.

3.3 Firms

Within each sector the structure of the model is akin to a standard NK model with a few modifications implied by the type of production functions. Each individual sector $i = 1, \ldots, N$ is composed by a continuum of atomistic-infinitely-lived firms indexed by $j \in [0, 1]$. Given a sector i, each firm j produces with the same constant returns to scale (CRS) production function,

$$Y_{(i,j),t} = Z_{i,t}^{p} L_{(i,j),t}^{\delta} \prod_{i'=1}^{N} X_{(i,j),i',t}^{\omega_{i,i'}}, \tag{9}$$

where $z^p_{i,t} := \log\left(Z^p_{i,t}\right)$ is a sectoral productivity shock following the AR(1) process

$$z^p_{i,t} = \rho_i z^p_{i,t-1} + \varepsilon^p_{i,t}, \tag{10}$$

$L_{(i,j),t}$ is labor used by firm (i,j) and $X_{(i,j),i',t}$ is a CES composite good made from products of all firms in industry i',

$$X_{(i,j),i',t} := \left(\int_0^1 X_{(i,j),(i',j')}^{\frac{\epsilon-1}{\epsilon}} dj'\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{11}$$

where the elasticity of substitution ϵ is the same across sectors and it is assumed to be the same as in the sectoral consumption bundles. Note that the share of a sectoral bundle i' in a firm's production in sector i is given by $\omega_{i,i'}$, and that the CRS assumption implies $\sum_{i'=1}^{N} \omega_{i,i'} = 1-\delta$. The N-by-N matrix containing these shares gives the representation of the production network and is denoted as $\mathbf{\Omega}$.

Because of the assumption of CRS production functions, Equation(9) implies all firms within a given sector face the same marginal cost. Thus, the firms' problem can be solved first by finding the optimal mix of inputs for a given output price and then finding the optimal price given the marginal cost of production with that input mix. This is what I do below.

3.3.1 Marginal cost

Appendix (A.1) shows that a firm (i,j)'s cost minimization problem can be written in terms of sectoral variables. A firm (i,j)'s cost of producing a given level of output $Y_{(i,j),t}$ is given by

$$\mathcal{C}\left(Y_{(i,j),t}\right) = \min_{L_{(i,j),t},\left\{X_{(i,j),i',t}\right\}} W_{i,t} L_{(i,j),t} + \sum_{i'=1}^{N} X_{(i,j),i',t} P_{i',t}$$

subject to the production function (9). The price index from an industry i' is defined as $P_{i',t} := \left(\int_0^1 P_{(i',j'),t}^{1-\epsilon} dj'\right)^{\frac{1}{1-\epsilon}}$. Note that the assumption of same elasticity of substitution in consumption and production allows for the definition of just one sectoral price index for both production and consumption. The solution to this problem yields the following formula for the nominal marginal cost of production in industry i,

$$MC_{i,t} = \frac{P_{i,t}^{m}\left(\frac{W_{i,t}}{\delta}\right)^{\delta}}{Z_{i,t}^{p}}, \tag{12}$$

where $P_{i,t}^{m}$ is the industry-specific price index of materials, defined by

$$P_{i,t}^{m} = \prod_{i'=1}^{N}\left(\frac{P_{i',t}}{\omega_{i,i'}}\right)^{\omega_{i,i'}}.$$

The CRS assumption allows me to write the total cost of production simply as:

$$\mathcal{C}\left(Y_{(i,j),t}\right) = MC_{i,t}Y_{(i,j),t}. \tag{13}$$

It is useful to produce expressions for the conditional factor demands of a particular firm. Given a level of output $Y_{(i,j),t}$, the nominal marginal cost and inputs prices, a firm (i,j) chooses labor and materials from industry i' such that

$$L_{(i,j),t} = \delta\frac{MC_{i,t}}{W_{i,t}}Y_{(i,j),t}, \tag{14}$$

$$X_{(i,j),i',t} = \omega_{i,i'}\frac{MC_{i,t}}{P_{i',t}}Y_{(i,j),t}. \tag{15}$$

3.3.2 Price optimization

Before presenting the price optimization problem it is necessary to derive the total demand for goods produced by a firm (i,j). A particular firm (i,j) can sell its product either as consumption good or as intermediate input to all firms from (possibly) all sectors. Thus, the market clearing condition faced by this firm is

$$Y_{(i,j),t} = C_{(i,j),t} + \sum_{i'=1}^{N}\int_{0}^{1} X_{(i',j')(i,j),t}dj'. \tag{16}$$

Appendix A.1.2 shows that, by using the optimality conditions implied by the CES structure of the consumption and production aggregators, equation (16) becomes

$$Y_{(i,j),t} = \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} Y_{i,t}, \tag{17}$$

where $Y_{i,t}$ represents total sectoral output and is defined as:

$$Y_{i,t} := C_{i,t} + \sum_{i'=1}^{N} X_{i',i,t}, \tag{18}$$

and where $X_{i',i,t} := \int_0^1 X_{(i',j'),i,t} dj'$ is the total demand from industry i' for inputs from industry i.

The problem of the firm is then to choose the optimal price $P_{(i,j),t}$ to maximize the expected present value of future profits,

$$\mathbb{E}_t \sum_{k=0}^{\infty} \frac{\Lambda_{t,t+k}}{P_{t+k}} \theta_i^k \left(P_{(i,j),t} - MC_{i,t+k}\right) Y_{(i,j),t+k}, \tag{19}$$

subject to the individual firm demand (17), and where $\Lambda_{t,t+k}$ is an endogenous discount factor between periods t and $t+k$.

The first-order condition of this problem allows to solve for the optimal sectoral price relative to the aggregate price index, $p^*_{i,t} := \frac{P^*_{i,t}}{P_t}$,

$$p^*_{i,t} = \frac{\epsilon}{\epsilon - 1} \frac{\psi_{i,t}}{\Delta_{i,t}}, \tag{20}$$

where $\psi_{i,t}$ and $\Delta_{i,t}$ are auxiliary variables representing the expected discounted value of marginal costs and marginal revenues respectively. Appendix A.1.3 shows that the expected discounted value of marginal costs follows the recursive relationship

$$\psi_{i,t} = Y_{i,t} C_t^{-\sigma} mc_{i,t} + \theta_i \beta \mathbb{E}_t \left[\Pi^{\epsilon}_{i,t+1} \psi_{i,t+1}\right], \tag{21}$$

where $\Pi_{i,t+1} := \frac{P_{i,t+1}}{P_{i,t}}$ is the sectoral gross inflation rate, and $mc_{i,t} := MC_{i,t}/P_t$ is the real marginal cost in this industry.

Appendix A.1.3 also shows that $\Delta_{i,t}$ can be expressed in terms of other variables to simplify the equilibrium. This produces the following relationship:

$$\left(\frac{1-\theta_i\Pi_{i,t}^{\epsilon-1}}{1-\theta_i}\right)^{\frac{1}{\epsilon-1}}\psi_{i,t} \;=\; \frac{\epsilon-1}{\epsilon}Y_{i,t}C_t^{-\sigma}p_{i,t}+\theta_i\beta\mathbb{E}_t\left[\Pi_{i,t+1}^{\epsilon-1}\left(\frac{1-\theta_i\Pi_{i,t+1}^{\epsilon-1}}{1-\theta_i}\right)^{\frac{1}{\epsilon-1}}\psi_{i,t+1}\right]. \tag{22}$$

Equations (21) and (22) are valid for every industry i and are required to solve the equilibrium system.

4 Equilibrium

In this section I describe the model's equilibrium system and some of its properties related to within-sector and between-sector price dispersion. The system of equations describing the equilibrium is composed by the optimality conditions of the household and the firms described in the previous section along with the market clearing conditions. The resulting system is then log-linearized around a steady state with trend inflation $\overline{\Pi}$. I start this section by describing the market clearing conditions and how these conditions are affected by within-sector price dispersion and the production network.

4.1 Market clearing

First, let us define the measure of within-sector price dispersion as $s_{i,t} := \int_0^1 \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} dj$. Appendix B shows that $s_{i,t}$ follows a backward looking law of motion depending on sectoral inflation and the degree of price stickiness,

$$s_{i,t} \;=\; (1-\theta_i)\left(\frac{1-\theta_i\Pi_{i,t}^{\epsilon-1}}{1-\theta_i}\right)^{\frac{\epsilon}{\epsilon-1}}+\theta_i\Pi_{i,t}^{\epsilon}s_{i,t-1}. \tag{23}$$

In a one-sector model this measure is simply referred to as "price dispersion" and is not relevant up to first-order approximations of the equilibrium around a zero-inflation steady state. Since firms are homogeneous and they whould all charge the same price without price frictions, this measure of price dispersion also represents a measure of price distortions. In a multi-sector model with heterogeneity, each sector has their own measure of price dispersion depending on sectoral price stickiness θ_i and the sectoral gross inflation rate $\Pi_{i,t}$. Most importantly, within-sector price dispersion creates inefficiencies, and in a multi-sector model these inefficiencies are transmitted to other sectors through the production network.

Within-sector price distortion inefficiencies. The type of inefficiencies caused by within-sector price distortion becomes clearer when obtaining the different market-clearing conditions of the model. Using the input demands of the firm (14) and (15) plus the output demand yields the following sectoral input demands:

$$L_{i,t} := \int_0^1 L_{(i,j),t} dj \quad \Rightarrow \quad L_{i,t} = \delta \frac{mc_{i,t}}{w_{i,t}} s_{i,t} Y_{i,t} \tag{24}$$

$$X_{i,i',t} := \int_0^1 X_{(i,j),i',t} dj \quad \Rightarrow \quad X_{i,i',t} = \omega_{i,i'} \frac{mc_{i,t}}{p_{i',t}} s_{i,t} Y_{i,t} \tag{25}$$

where $w_{i,t} := W_{i,t}/P_{i,t}$ represents the real sectoral wage.

These sectoral input demands give a partial preview of within-sector price dispersion costs; keeping everything else constant, higher price dispersion increases the quantities of inputs needed for the same quantity of output.

This is a partial analysis, however, because within-sector price dispersion also affects the real marginal cost of each sector. To visualize this issue, we can use the marginal rate of substitution (3) and condition (7) to write the sectoral labor supply in terms of the real sectoral wage, $\frac{L_{i,t}^{\varphi}}{C_t^{-\sigma}} = w_{i,t}$. Setting labor supply equal to labor demand in sector i yields the condition:

$$\frac{w_{i,t}}{\delta} = \left[\frac{p_{i,t}^m s_{i,t} Y_{i,t}}{\delta^{\frac{1}{\varphi}} Z_{i,t}^p} \right]^{\frac{\varphi}{1+(1-\delta)\varphi}} C_t^{\frac{\sigma}{1+(1-\delta)\varphi}},$$

which can be replaced in the marginal cost eliminate the sectoral wage,

$$mc_{i,t} \quad = \quad \left(\frac{p_{i,t}^m}{Z_{i,t}^p} \right)^{\frac{1+\varphi}{1+(1-\delta)\varphi}} \left(\frac{s_{i,t} Y_{i,t}}{\delta^{\frac{1}{\varphi}}} \right)^{\frac{\delta\varphi}{1+(1-\delta)\varphi}} C_t^{\frac{\delta\sigma}{1+(1-\delta)\varphi}}. \tag{26}$$

Again, a partial analysis shows that higher values of $s_{i,t}$ increase the real marginal cost of production, making sectors less efficient.

These sectoral inefficiencies negatively affect other industries through the production network channel. To see this, we can use the sectoral input demands (25), the sectoral consumption condition $C_{i,t} = \alpha_i \frac{C_t}{p_{i,t}}$ and the market-clearing condition (18) to get

$$Y_{i,t} = \alpha_i \frac{C_t}{p_{i,t}} + \sum_{i'=1}^{N} \omega_{i',i} \frac{mc_{i',t}}{p_{i,t}} s_{i',t} Y_{i',t}. \tag{27}$$

The first term of the right-hand side in Equation (27) corresponds to sectoral consumption demand, while the second term represents the total demand for inputs from industry i. Evaluating the latter equation for every sector i defines a system that determines the set of gross sectoral production $\{Y_{i,t}\}_{i=1}^{N}$ as a function of the other endogenous variables C_t, $\{p_{i,t}, s_{i,t}\}_{i=1}^{N}$ and the exogenous shocks, $\{Z_{i,t}^{p}\}_{i=1}^{N}$. Price dispersion in other sectors is transmitted through the need of intermediate inputs given by the shares $\omega_{i,i'}$.

4.2 Model solution and relative price gaps.

The system determining the solution for the model consists of two aggregate equations and $5N$ sectoral equations to determine $5N+2$ variables: value-added output C_t, the nominal interest rate I_t, and the sectoral variables $\{\Pi_{i,t}, s_{i,t}, \psi_{i,t}, Y_{i,t}, p_{i,t}\}$. The two aggregate equations are the Euler equation of the household (4) plus the Taylor rule (8). The sectoral equations are given by the optimal pricing equations (21) and (22), the law of motion of within-sector price distortion (23), the market clearing condition (27) where real marginal costs can be replaced using (26), and the following identity determining the evolution of sectoral relative prices,

$$p_{i,t} = \frac{\Pi_{i,t}}{\Pi_t} p_{i,t-1}. \tag{28}$$

The log-linearized system of equations is reported in Appendix B.2. In this type of steady state, every sector's nominal price keeps growing at a (potentially different) gross inflation rate $\overline{\Pi}_i$, which generates an aggregate gross inflation rate $\overline{\Pi} = \prod_{i=1}^{N} \overline{\Pi}_i^{\alpha_i}$. An important feature of the log-linearized system is the presence of relative price gaps, that is the log-difference between the relative price of a sector and its steady state level $\hat{p}_{i,t} := \log\left(\frac{P_{i,t}}{P_t}\right) - \log\left(\frac{P_i}{P}\right)$. The effects of these distortions are analyzed in Section 6.

The next section studies some of the properties of the steady state equilibrium.

4.3 Steady state equilibrium

Although the solution of a positive inflation steady state is not available without resorting to numerical methods, in this section I briefly highlight some key features of such a steady state.

Steady state distortions of nominal frictions. The main difference between a zero-inflation steady state and one with positive long-run inflation is the importance of within-sector price dispersion. When inflation is zero in the long run, price dispersion becomes irrelevant in a first-order approximation because the model is approximated exactly at the point where price dispersion become minimal. On the other hand, when long-run inflation is positive, price dispersion becomes non-negligible in a first-order approximation.

Evaluating equation (59) in steady state yields the long run level of price dispersion:

$$\overline{s}_i \;=\; \frac{1-\theta_i}{1-\theta_i \Pi_{i,t}^{\epsilon}} \left(\frac{1-\theta_i \overline{\Pi}_i^{\epsilon-1}}{1-\theta_i}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{29}$$

Sectoral steady-state price dispersion is increasing with the degree of price stickiness θ_i, the elasticity of substitution ϵ, and sectoral trend inflation $\overline{\Pi}_i$. Note that without trend inflation, the gross inflation rate is $\overline{\Pi}_i = 1$ and $\overline{s}_i$ attains its minimum at $\overline{s}_i = 1$, which is illustrated in Figure 2a. Furthermore, the price dispersion measure is not well defined when $\left(\frac{1}{\theta_i}\right)^{\frac{1}{\epsilon}} < \overline{\Pi}_i$ since this would result in negative values. As a result the existence of an equilibrium is restricted by this condition, which must be satisfied in every sector. Figure 2b illustrates this fact. The figure shows that given the elasticity of substitution ϵ, higher degrees of price stickiness θ_i restrict the possible values of trend inflation over which $\overline{s}_i$ is well defined. This condition makes models with heterogeneous price stickiness less stable, something that is discussed later.

Another interesting feature comes from evaluating the real marginal cost in steady state,

$$\overline{mc}_i \;=\; (\overline{p}_i^m)^{\frac{1+\varphi}{1+(1-\delta)\varphi}} \left(\frac{\overline{s}_i \overline{Y}_i}{\delta^{\frac{1}{\varphi}}}\right)^{\frac{\delta\varphi}{1+(1-\delta)\varphi}} \overline{C}^{\frac{\delta\sigma}{1+(1-\delta)\varphi}}. \tag{30}$$

This expression shows that given a particular value for the other variables, higher price dispersion in a particular sector increases the marginal cost of production in that sector. Note without trend inflation $\overline{\Pi}_i = 1$ and thus $\overline{s}_i = 1$ so that the marginal cost is not affected. This result has been studied before by other authors (e.g. Ascari and Sbordone, 2014) , who show the link between higher price dispersion and lower steady state consumption. What is new here is the fact that – due to the existence of production networks – price dispersion in sector i not only affects this sector's marginal cost of production, but also increases the cost of all other sectors using sector i's goods as inputs for their own productive process. This is explored below by mapping the multi-sector model into a one-sector economy with an aggregate production function.

4.3.1 Aggregate Production Function

To illustrate the effects of long-run inflation, sectoral heterogeneity, and production networks in this subsection I show that the N-sector model with constant returns to scale is isomorphic with a one-sector economy with a production function linear in aggregate labor. Total factor productivity of this production function (measured as the Solow residual) is an endogenous object depending upon, among other things, sectoral trend inflation and the structure of the production network. The procedure to generate this aggregate production function is similar to the one in Jones (2011) with the difference of including monetary frictions and monopolistic competition of firms.

Before showing the aggregate production function I need to define some endogenous objects of the model. Let

$$\eta_{i,t} : \;\; = \;\; \frac{Y_{i,t} p_{i,t}}{C_t} \tag{31}$$

denote the Domar weight of sector i, that is the ratio of the value of sectoral production i relative to aggregate GDP. Appendix B.3.1 shows how to determine the Domar weights given the endogenous variables of the model. Next, let

$$\kappa_{i,t} := L_{i,t}/L_t \tag{32}$$

denote the share of sectoral labor $L_{i,t}$ relative to the aggregate labor index. Appendix B.3 shows that in equilibrium this share is a function of the Domar weights and other endogenous variables of the model. With this information I can now state Proposition 1.

Proposition 1. *Under the assumptions of the model presented in Section 3.1, Section 3.3 and Section 4, and given the definitions (31) and (32), the multi-sector economy with production networks and nominal frictions is isomorphic with a one-sector economy with an aggregate production function linear in aggregate labor. In steady state the aggregate production function satisfies*

$$\overline{C} = \overline{TFP} \cdot \overline{L} \tag{33}$$

where

$$\overline{TFP} = exp\left\{\boldsymbol{\alpha}' \left[\overline{\tilde{\boldsymbol{\alpha}}} + (\mathbf{I} - \boldsymbol{\Omega})^{-1} \overline{\mathbf{a}}\right]\right\} \tag{34}$$

where $\overline{\tilde{\boldsymbol{\alpha}}}$ and $\overline{\mathbf{a}}$ are N-by-1 vectors with typical element defined by

$$\overline{a_i} := \delta log\left(\overline{\kappa}_i\right) + \sum_{i'=1}^{N} \omega_{i,i'} log\left(\omega_{i,i'} \frac{\overline{mc}_i \overline{s}_i}{\overline{p}_i} \frac{\overline{\eta}_i}{\overline{\eta}_{i'}}\right) \tag{35}$$

$$\overline{\tilde{\alpha}_i} := log\left(\frac{\alpha_i}{\overline{\eta}_i}\right). \tag{36}$$

For a proof of Proposition 1 please refer to Section B.3.2 of the Appendix.

I close this section noting that Proposition 1 together with the steady state conditions (29) and (30) shows that trend inflation and heterogeneity of price stickiness affect $\overline{TFP}$ directly through marginal costs $\overline{mc}_i$ and within-sector price dispersion $\overline{s}_i$. These monetary frictions also affect $\overline{TFP}$ indirectly through the steady state equilibrium levels of the other endogenous variables $\overline{p}_i$, $\overline{\kappa}_i$ and $\overline{\eta}_i$. $\overline{TFP}$ also depends on the Leontief inverse $(\mathbf{I} - \boldsymbol{\Omega})^{-1}$, showing how production networks can play a role amplifying distortions. I return to this discussion in Section 7 to understand the quantitative effects these different elements have on $\overline{TFP}$ and welfare.

5 Calibration

This section describes the baseline calibration of the model and the data sources. One of the objectives of the calibration is to compute the model's dynamic simulations in the presence of monetary and productivity shocks. To this end, I use the sectoral definitions of the 35-sector

$KLEMS$ dataset developed by Dale W. Jorgenson and described in Jorgenson et al. (2000). The dataset covers 35 sectors at roughly the 2-digit SIC level from 1960 to 2005, however I exclude the government sector to match the model. As explained in Section 5.1 and below, the $KLEMS$ dataset provides detailed sectoral data of gross output plus quantities and prices of inputs used in each industry's production. This allows for the calibration of the input-output structure of the 34-sector economy and the estimation of the process of sectoral productivity. This procedure is detailed in Section 5.2 below. For the frequency of price changes I use the calibration from Pasten et al. (2016), which is constructed using confidential microdata underlying the producer price index (PPI) from the BLS. These data are described in Section 5.3. The rest of the parameters are calibrated using standard business cycle values and are described in Section 5.4. The latter section also describes 5 different economies that will be studied in the rest of the paper.

5.1 Productivity shocks

For each sector, the $KLEMS$ dataset contains yearly information of quantities and prices of sectoral output and inputs (capital $K_{i,t}$, labor $L_{i,t}$ and materials $M_{i,t}$). I calibrate the parameters of the following sectoral production function:

$$Y_{i,t} \;=\; K_{i,t}^{\delta_i^k} L_{i,t}^{\delta_i^l} M_{i,t}^{1-\delta_i} Z_{i,t}, \tag{37}$$

by computing time-averages of the cost shares for capital, labor, and materials. I calibrate these shares by dividing the corresponding input cost by the total cost of production. Given these shares, sectoral productivity shocks, $Z_{i,t}$, are computed as the residuals of Equation (37). The stochastic process of the detrented version of these shocks is estimated according to Equation (10), and a time series for the sectoral residuals $\{\varepsilon_i^p\}_{i=1}^N$ is produced. I estimate the variance-covariance matrix $\Sigma := \mathbb{E}\left[\boldsymbol{\varepsilon}_t^p \left(\boldsymbol{\varepsilon}_t^p\right)'\right]$ using the time series of these sectoral residuals. Table 1 presents the estimated autoregressive coefficients for each sector, as well as the estimated standard deviation of the residuals and the calibrated share of intermediates inputs, $1-\delta_i$. In the model simulations I set the share of intermediate inputs equal to the average share of intermediate inputs across sectors, which is approximately 0.45, and the share of labor equal to 0.55 to maintain CRS in the production functions.

5.2 Production network

The *KLEMS* dataset provides details about industry use of materials that can be used to calibrate the economy's production network. For every industry i, the dataset contains the quantities of sectoral commodities used in production that year, $X_{i,i'}$, and its sectoral price index, $P_{i'}$. This allows me to construct cost shares for each sectoral input. On the other hand, equation (25) can be mapped into the data to identify the coefficients of the production network since it implies that the cost share of a sectoral input relative to the total cost of materials is:

$$\frac{X_{i,i',t}P_{i',t}}{\sum_{i'=1}^{N} X_{i,i',t}P_{i',t}} = \frac{\omega_{i,i'}}{(1-\delta)}. \tag{38}$$

Given the share of intermediate inputs $1-\delta$, equation (38) identifies the coefficients in $\boldsymbol{\Omega}$. I calibrate these coefficients by constructing the cost shares and taking the time-average for each industry. The resulting input-output matrix is illustrated in Figure 2. This figure shows that the diagonal elements of the input-output matrix tend to be the most important ones, and only few sectors are important producers of inputs used by other industries. To have a measure of sector centrality in the supply chain, I define the weighted out-degree of a sector i as the share of i's output in the input supply of the entire economy, $O_i := \sum_{i'=1}^{N} \omega_{i',i}$. The last column of Table 1 reports the weighted out-degree for each sector. The most important industries in the 34-sector economy are Services, Trade, Chemicals, and Finance Insurance and Real State.

5.3 Frequency of price changes

To calibrate the frequency of price change in the 34-sector economy I use data from Pasten et al. (2016). These authors calibrate monthly frequencies of price changes using confidential microdata underlying the BLS's Producer Price Index (PPI). They calculate the frequency of price adjustments at the goods level and aggregate these goods-based frequencies to a 350-sector industry level. This is done according to the industry definitions of the Bureau of Economic Analysis (BEA), which maps approximately to the 4-digits NAICS code aggregation level. I produce the 34-sector frequencies by: (i) mapping the 4-digit NAICS codes to the 2-digit SIC codes using the guidelines provided by the BEA, and (ii) by computing the median frequency within each 2-digit SIC code. This procedure yields an average monthly frequency of 21.1%, corresponding to an implied duration of prices of 4.2 months. Finally, since in the rest of the paper the unit of time is quarterly, I express the frequencies of price change computed in this section also by quarter.

5.4 Other parameters

Business cycle parameters. The rest of the model's parameters are set equal to standard values from the business cycle literature. The unit of time is a quarter so the subjective discount factor is set equal to $\beta = (0.967)^{\frac{1}{4}}$ to target the average of the 1-Year Treasury Constant Maturity Rate between 1962 and 2016. The coefficient of relative risk aversion is $\sigma = 1$, and the inverse of the Frisch elasticity of labor is $\varphi = 1$. The parameters of the Taylor rule are $\rho_I = 0.8$, $\phi_c = 0.5/4$ and $\phi_\Pi = 2$ similar to those estimated in Smets and Wouters (2007). The autoregressive coefficient and quarterly standard deviation of the aggregate productivity shock and aggregate monetary shock are $\rho_p = 0.95$, $\sigma_p = 0.45$, $\rho_m = 0.15$ and $\sigma_m = 0.24$, respectively (as estimated in Smets and Wouters 2007). Finally the constant elasticity of substitution is set equal to $\epsilon = 7$. Table 2 summarizes the values of these parameters.

Five different economies. I use the baseline calibration described above to compare a standard one-sector model with a multi-sector economy with production networks and heterogeneous frequencies of price change. To build an intuition of how different dimensions of sectoral heterogeneity affect the results, I introduce three intermediate cases, each one seeking to highlight a key feature of the model. Table 3 reports the details of the 5 cases, but I state the main characteristics below:

Case 1: one-sector model, no production networks, homogeneous frequencies of price change

Case 2: multi-sector model, no production networks, homogeneous frequencies of price change

Case 3: multi-sector model, no production networks, heterogeneous frequencies of price change

Case 4: multi-sector model, production networks, homogeneous frequencies of price change

Case 5: multi-sector model, production networks, heterogeneous frequencies of price change

Case 1 is the standard NK model. Case 2 only increases the number of sectors. Case 3 introduces differences in price stickiness across sectors. Case 4 is similar to Case 2 but it adds input-output links between sectors. Case 5 combines the different types of heterogeneity.

6 Inflation Dynamics

In this section I discuss how the multi-sector model with heterogeneity compares to the standard one-sector model in terms of inflation sensitivity with respect to aggregate output gap and inflation inertia. The sensitivity of inflation to the output gap is important for many reasons. First, from a policy perspective, inflation sensitivity to the output gap gives a sense of how does real activity affects inflation. For example, given a positive output gap, higher sensitivity would mean higher inflationary pressures and would prompt monetary authorities to raise interest rates more quickly. Secondly, inflation sensitivity to the output gap is inversely related to the magnitude and persistence of the real effects of monetary shocks.[6] When inflation is less sensitive to movements in real activity and thus aggregate price adjustments take longer to occur, the slower adjustment feeds back to aggregate output, adding sluggishness to the convergence process. On the other hand, inflation inertia has similar implications in terms of slowing the price adjustment process and amplifying the real effects of monetary shocks. The backward-looking nature of within-sector price dispersion (Equation (23)) and relative sectoral prices (Equation (28)) adds inertia to the equilibrium system. Although the within-sector measure of price dispersion is also present in a one-sector model with trend inflation, relative sectoral prices are, by definition, only relevant in a model with more than one sector.

6.1 The generalized New Keynesian Philips Curve

To give a measure of the sensitivity of inflation to the output gap, I derive the generalized NKPC. I call the equation linking inflation to output gap the "generalized NKPC" because it embeds several versions of the NKPC found in the literature.[7] Appendix C.1 shows how to produce the NKPC from the log-linearized equilibrium system. I also derive the NKPC of two special cases to highlight the main differences with the standard one-sector model.

Generalized NKPC. The generalized NKPC from the model laid out in Section 3 and Section 4 is given by

$$\hat{\Pi}_t = \Phi_c \hat{C}_t + \sum_{i=1}^{N} \left[\Phi_i^p \hat{p}_{i,t} + \Phi_i^s \hat{s}_{i,t} + \beta \Phi_i^\pi \mathbb{E}_t \left[\hat{\Pi}_{i,t+1} \right] + \beta \Phi_i^\psi \mathbb{E}_t \left[\hat{\psi}_{i,t+1} \right] + \Phi_i^z z_{i,t}^p \right], \tag{39}$$

[6] See Woodford (2011) Chapter 3 for a detailed discussion of the one-sector model.

[7] Sections C.1.1 to C.1.4 of the Appendix show how different versions of the NKPC can be obtained as special cases of the generalized NKPC.

where variables with hat denote log-deviations with respect to their steady state levels, i.e. $\hat{X}_t := \log X_t - \log \overline{X}$. The coefficients Φ_c, $\left\{\Phi_i^{\pi}, \Phi_i^{\psi}, \Phi_i^{p}, \Phi_i^{s}, \Phi_i^{z}\right\}_{i=1}^{N}$ depend only on parameters of the model and are defined in Appendix C.1. Equation (39) shows that, contrary to the standard NKPC, aggregate inflation depends on a number of endogenous variables other than future expected inflation and the aggregate output gap. Expectations of the present value of future sectoral marginal costs $\mathbb{E}_t\left[\hat{\psi}_{i,t+1}\right]$, relative sectoral price gaps $\hat{p}_{i,t}$ and sectoral price dispersion gaps $\hat{s}_{i,t}$ are all endogenous variables that also affect aggregate inflation and are ignored in a simpler model.

There are several implications from the generalized NKPC in (39). First is the coefficient of the output gap changes relative to the one-sector economy. This is analyzed below. Second is the log-linearized version of equations (23) and (28) include the variables $\{\hat{p}_{i,t}, \hat{s}_{i,t}\}_{i=1}^{N}$ into the aggregate NKPC adds inertia to the inflationary process since these variables are backward looking (see equations (59) and (61) in Appendix B.2). Third, the heterogeneity of the model implies that some sectors are relatively more important in determining inflation. Finally, the presence of endogenous variables other than the output gap can bias the estimated coefficients of a standard NKPC if the omitted variables are correlated with those included in the estimation.

NKPC under no trend inflation. To build some intuition about the magnitude of the coefficient Φ_c equation (40) presents the NKPC in the case with no trend inflation,[8] i.e. $\overline{\Pi}_i = 1$. Without trend inflation the coefficients of future expected marginal costs Φ_i^{ψ} converge to zero while the within-sector price dispersion measures $\hat{s}_{i,t}$ are no longer relevant in a first-order approximation of the system.

$$\hat{\Pi}_t = \Phi_c \hat{C}_t + \beta \mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] + \sum_{i=1}^{N}\left[\Phi_i^{p}\hat{p}_{i,t} + \Phi_i^{z} z_{i,t}^{p}\right] \tag{40}$$

In this case the coefficients $\{\Phi_i^{p}, \Phi_i^{z}\}_{i=1}^{N}$ are different from the values taken in (39) and are defined in Appendix C.1.1. Note that relative price gaps are still present in the NKPC so even under zero trend inflation the equation includes the backward-looking variables.

[8] This statement is about sectoral trend inflation. Technically one could have a case where sectors have different trend inflation rates that cancel out in the aggregate. Equation (40) would not be valid in that case.

Standard NKPC The standard NKPC can be produced from (40) by assuming $N = 1$ and setting $\delta = 1$ to keep the constant returns to scale. Under these assumptions the NKPC becomes

$$\hat{\Pi}_t = \Phi_c \hat{C}_t + \beta \mathbb{E}_t \left[\hat{\Pi}_{t+1} \right] + \Phi^z z_t^p, \tag{41}$$

and by definition there are no sectoral price gaps. The next section compares the coefficients of the output gap.

6.1.1 The sensitivity of inflation towards the aggregate output gap

The value of the coefficient Φ_c varies depending on the parameters of the model. Under the more general case of equation (39), Φ_c involves weighted averages of sectoral inflation trends, price stickiness and measures of sector centrality. The precise formula is presented in Appendix C.1.

Figure 3a displays the coefficient Φ_c for the standard one-sector economy and the multi-sector economy for 3 different values of trend inflation. The figure shows two important characteristics. For each case, Φ_c is decreasing with the level of trend inflation, implying the inflation gap reacts less to a given level of the output gap. Second, for any level of trend inflation the multi-sector economy always displays a lower value of Φ_c. Figure 3b compares the value of Φ_c for the 5 cases described in Table 3. Case 3 in the figure shows that adding only the heterogeneity of price stickiness increases Φ_c, making inflation more sensitive to output-gap variations. However Case 4 shows the isolated effect of adding production networks: Φ_c at about 60% compared to the one-sector economy of Case 1. The final effect of these two forces can be observed in Case 5. At an inflation trend of 2%, the coefficient of Case 5 is about 30% smaller than that of Case 1.

6.1.2 Sectoral NKPC coefficients

The generalized NKPC of equation (39) includes several sectoral variables. In this section I briefly analyze some of the coefficients of these variables. For this analysis Figures (4) and (5) report a scatter plot of the coefficients of relative price gaps and sectoral price dispersion gaps, respectively. Table (4) reports correlations between sectoral coefficients and sectoral measures of price stickiness and network centrality. These figures and table are useful to isolate key characteristics of each type of coefficient, which are discussed below.

Relative price gap coefficients, Φ_i^p. First we have the coefficients associated with sectoral price gaps, $\{\Phi_i^p\}$. Equation (40) shows that sectoral price gaps are also relevant in a setting without trend inflation. Since sectoral price gaps are defined as the log-difference of relative sectoral prices with respect to their steady state level, these variables are only relevant in multi-sector models. Figure 4 displays the coefficients Φ_i^p for the economies described in Cases 2 to 5 when there is no trend inflation. In Case 2 every sector has a negative coefficient equal to -0.02, meaning that a positive sectoral price gap of 1% would decrease inflation by 2bp, reflecting expectations of that sector closing its gap in the future. Compared to Case 2, Case 3 only introduces heterogeneity in the frequency of price stickiness. Figure 4a shows that heterogeneity increases dispersion of the coefficients, and columns (a) and (b) of Table 4 indicate that more flexible sectors (i.e. sectors with lower θ_i) have negative coefficients with bigger absolute value.

To isolate the effect of having production networks, we can compare Case 2 and Case 4 in Figure 4b. Just by adding production networks, the coefficients Φ_i^p increase, some of them even taking positive values. The change in sign may be due to a substitution of a sector's output in the production chain; a positive sectoral price gap would make other sectors substitute inputs normally bought from the sector with higher prices, increasing the current cost of production and hence inflation. The positive correlation in column (a) of Table 4 shows that this effect is more important for more central sectors.

The total effects of these forces are reflected in the coefficients of Case 5 in Figure 4. In the calibrated economy of Case 5, the magnitude of the negative coefficients is dampened relative to Case 3, while the magnitude of some positive coefficients is amplified relative to Case 4. Tables 5 and 6 show the coefficients for the calibrated economy of Case 5, when trend inflation is 0% and 2% respectively. Agriculture and Metal Mining are the most important sectors with negative price coefficients. Without trend inflation Table 5 shows that a positive 1% price gap in those sectors would trigger a decrease in inflation of 22bp and 26bp in the agriculture and metal mining sectors, respectively.

Within-sector price dispersion gap coefficients, Φ_i^s. An2other important set of coefficients is that associated with within-sector price dispersion gaps, $\{\Phi_i^s\}$. Figure 5 shows these coefficients in decreasing order when trend inflation is zero. The coefficients are positive for all the cases analyzed, implying that positive (negative) within-sector price dispersion gaps trigger increases (decreases) in aggregate inflation. Comparing Case 2 and Case 3 in Figure 5a shows the effect of heterogeneity of price stickiness. Again, the heterogeneity of Case 3 amplifies the dispersion of the coefficients. Columns (c) and (d) of Table 4 show that this is due to sectors with more flexible prices having higher coefficients. On the other hand, comparing Case 2 and Case 4 in Figure 5b, one can see that the effect of adding production networks dampens the magnitude of the coefficients. Although more central sectors have higher coefficients according to columns (c) and (d) in Table 4, their magnitude is small compared to Case 2. As a result the calibrated economy of Case 5 looks like a less extreme version of Case 3. Tables 5 and 6 show that again Agriculture and Metal Mining are the most relevant sectors. Without trend inflation, a 1% gap in within-sector price dispersion would cause an increase in inflation of 10 bp in the case of Agriculture and 9bp in the case of Metal Mining (Table 5).

Other sectoral coefficients. An analysis similar to the one above shows that the coefficients of sectoral productivity shocks are driven by forces similar to those determining the within-sector price dispersion coefficients. Positive productivity shocks decrease inflation, and they do so more in central sectors with more flexible prices (see Figure 6 and columns (e) and (f) in Table 4). Again, heterogeneity of price stickiness appears to be the main force driving quantitative differences across sectors. With respect to the other coefficients present in the generalized NKPC (39), Tables 5 and 6 show two things. First, there is no sectoral variation in the coefficients of future expected sectoral inflation Φ_i^π. This is mainly explained by assuming the same inflation trend and consumption shares across sectors. Second, the coefficients linked to discounted marginal costs, Φ_i^ψ are quantitatively not important. Finally, by comparing the summary statistics at the bottom of Tables 5 and 6, we see that increases in trend inflation decrease the overall importance of Φ_i^p, Φ_i^s, and Φ_i^z while increasing the overall importance of Φ_i^π, Φ_i^ψ. Thus the NKPC becomes more forward looking as trend inflation increases.

6.2 Impulse response functions

The analysis of the previous section provides some suggestive evidence of what differences should be expected when comparing the dynamic properties of inflation in a one-sector model with a multi-sector one. Compared to a one-sector model, inflation in a multi-sector model reacts less to variations in aggregate output and should have more inertia because of the presence of relative sectoral price gaps, which follow a backward looking process. This analysis is, however, a partial one since the output gap and sectoral variables such as relative price gaps are all endogenous variables of the dynamic equilibrium system. This section presents the impulse response functions of aggregate inflation and the aggregate output gap to different types of exogenous shocks.

6.2.1 Real effects of monetary shocks

To measure the real effects of monetary shocks Figure 7a compares the impulse response functions of the one-sector and the multi-sector economies to a 1% annualized monetary shock. Just as expected from the the analysis in Section 6.1.1, the real effects of monetary shocks are more important in the multi-sector case. Compared to the one-sector economy, inflation reacts less to the monetary shock in the multi-sector economy of Case 5 and as a result the response of the output gap is larger, increasing the degree of monetary non-neutrality. This can also be seen in Table 7 that shows the impulse response at impact and the cumulative effect after 4 quarters for both the output-gap and inflation.

Under the baseline calibration of the model, trend inflation does not play a large role in modifying these impulse response functions. The dashed lines of Figure 7a, representing the responses when trend inflation is 4%, are very similar to the solid lines, representing the responses when trend inflation is 0%.[9] Table 7 shows that impulse responses at impact and after 4 quarters are very similar across values of trend inflation. Maybe a more important consequence of higher trend inflation is the fact that, for values of trend inflation above 4%, the multi-sector model of Case 5 does not have a unique equilibrium. Since the same phenomenon holds true for the multi-sector economy of Case 3, this suggests that multiple equilibria are a consequence of adding heterogenous degrees of price stickiness into the model.

Finally, Figure 7b and Table 7 allow for the comparison of how different characteristics affect the responses of inflation and output gap to a monetary shock. Facing a monetary shock, the multi-sector economy of Case 2 displays the same dynamics of the one-sector economy. Since the economy is only facing an aggregate shock and, in Case 2 all sectors can adjust their prices at the same speed, having one or multiple sectors does not affect the dynamic results. On the other hand, Figure 7b shows that Cases 3, 4, and 5 display similar dynamic responses to the aggregate

[9]These results change depending on the calibration of the model. For example, higher values for the elasticity of substitution ϵ increase the degree of monetary non-neutrality in cases with larger trend inflation. For the baseline calibration the effects of trend inflation can be observed in the one-sector model at higher levels of this variable (see Figure 8). However, as discussed in this section, higher levels of trend inflation trigger multiple equilibria in the multi-sector economy of Case 5.

shock. Table 7 shows that the responses in Case 5 at impact and after the 4th quarter are mainly explained by the production network introduced in Case 4. For example, when trend inflation is 0% annually, the output gap drops 0.34% in the standard one-sector economy (Case 1) compared to 0.57% in the calibrated multi-sector economy (Case 5). Adding only price stickiness heterogeneity (Case 3) would produce a drop of 0.43% whereas adding only production networks (Case 4) would produce a drop of 0.50% , closer to the value observed in the calibrated multi-sector economy.

6.2.2 Inflation inertia

The presence of bakward-looking variables in the generalized NKPC adds inertia to the inflationary process. To illustrate this, Figure 9 displays the correlograms for Cases 1 and 5. The correlograms are estimated after solving each model and performing a MonteCarlo simulation with 10,000 repetitions of 180 periods each. The simulations assume productivity and monetary shocks according to the calibration in Section 5. Figure 9 displays the results without trend inflation. We can see that, compared to Case 1, the autocorrelation of inflation increases in the multi-sector model.

7 Trend inflation, welfare, and monetary policy

This section compares the welfare costs of trend inflation between the standard one-sector model and the multi-sector model with heterogeneity. To measure welfare costs I compute the consumption equivalent, CE, that is the fraction by which consumption must be increased in an equilibrium with price frictions as to produce the same expected utility of a flexible price equilibrium. The consumption equivalent is implicitly defined by:

$$\mathbb{E}\left[U\left(L_{\overline{\Pi}}, C_{\overline{\Pi}}\left(1+CE\right)\right)\right] \quad = \quad \mathbb{E}\left[U\left(L_F, C_F\right)\right], \tag{42}$$

where $L_{\overline{\Pi}}, C_{\overline{\Pi}}$ are optimal labor and consumption in an economy with price frictions and trend inflation equal to $\overline{\Pi}$, and L_F, C_F are optimal labor and consumption in the flexible price equilibrium. To compute the consumption equivalent I solve the model and perform a Montecarlo simulation with 10,000 repetitions of 180 periods each, which corresponds to the sample size used to calibrate the parameters of the productivity shocks in Section 5 . This procedure allows me to compute expected utility and solve for the consumption equivalent. In the section below, I solve and simulate the model for Cases 1 to 5 to analyze the components of welfare costs associated with trend inflation.

7.1 Welfare costs

Figure 10a displays welfare costs as a function of trend inflation for the one-sector economy of Case 1 and the calibrated multi-sector economy of Case 5. The differences are stark. Although both models predict similar costs when trend inflation is zero, the costs rise sharply in the multi-sector model. When trend inflation is 2% annually, the one-sector model displays a welfare cost of 0.03% compared to the welfare cost of 0.34% of the multi-sector model, increasing the losses by one order of magnitude. At a 4% level of trend inflation, the losses are 0.10% in the one-sector model versus 2.28% in the multi sector model. For values of trend inflation above 4% annually a unique equilibrium of the multi-sector model of Case 5 does not exists. I discuss this fact below in this section.

To shed a light on the sources of these differences, Figure 10a and Table 8 present the welfare costs for the all cases described in Table 3. Several things are worth noticing. First, and differently from the results of Section 6, the one-sector economy (Case 1) and the multi-sector economy without production networks and homogeneous price stickiness (Case 2) do not yield the same results. Although the impulse response functions of these two cases were almost the same when facing an aggregate shock (see Figure 8 and Table 7), the welfare costs of trend inflation are larger in Case 2. This difference is partly explained by the presence of sectoral shocks in the simulations of Case 2; even though all sectors can react at the same speed to aggregate shocks, the presence of sectoral shocks under nominal frictions distorts relative sectoral prices. The results for Case 3 show that adding heterogeneity in price stickiness to a multi-sector model without production networks amplifies these distortions even further. In fact, the distortions produced in Case 3 almost match those of the fully calibrated economy of Case 5. Furthermore, just as in Case 5, Case 3 does not have a unique equilibrium for trend inflation above 4%. This points to heterogeneity in price stickiness as the source of equilibrium instability as trend inflation increases. Finally, the case of a multi-sector economy with production networks but homogeneous price stickiness across sectors (Case 4), yields higher welfare losses than the homogeneous case without production networks. Thus production networks do amplify the welfare costs of trend inflation but do not represent the most important amplification factor.

7.1.1 Sources of welfare costs.

Steady state inefficiencies. To understand the sources of welfare costs Figure 11 illustrates the effect of trend inflation on steady state consumption and labor. Compared to the flexible price equilibrium, steady-state consumption drops sharply in the multi-sector economy of Case 5, whereas the decrease is much less pronounced in the standard one-sector model of Case 1. In Case 5, with trend inflation equal to 2%, steady-state consumption drops 0.25% with respect to its flexible price level. The same value is much lower for Case 1, where steady-state consumption drops 0.02% with respect to its flexible price level. On the other hand, steady-state labor increases more rapidly in Case 5 compared to Case 1 (0.11% versus 0.01% with trend inflation of 2%). As a result, with trend inflation of 2% aggregate TFP drops 0.36% in Case 5 compared to the 0.02% drop of Case 1.

From the analysis of Section 4.3.1 we know that TFP is a function of endogenous variables of the model and structural parameters. But how can we give a quantitative measure of the individual significance of these forces affecting TFP? We can compare the different cases in Figure 12 to answer this question. The figure shows that the multi-sector economy with homogeneous price stickiness and no production networks (Case 2) produces similar results than the standard one-sector economy. Compared to this case, Case 4 shows that introducing production networks does increase TFP losses, particularly at higher levels of trend inflation. However, what seems to be quantitatively crucial in creating productivity losses is introducing heterogeneity in the degree of price stickiness across different sectors (Case 3). Comparing TFP losses for Cases 3 and 5, we infer that production networks play only a minor role in amplifying TFP losses after accounting for heterogeneity of price stickiness.

It has been argued before that the within-sector price dispersion measure, $s_i = \int_0^1 \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} dj$, causes inefficiencies that mimic those of productivity losses (Yun, 2005) and that those inefficiencies become quantitatively more important with higher trend inflation (Ascari and Sbordone, 2014). The analysis in this section shows that losses caused by within-sector price distortions are only secondary to those coming from distortions changing sectoral relative prices.

Volatility increases. The second type of source of welfare costs is the effect that trend inflation has on the volatility of labor and consumption. Because of risk aversion, expected utility is decreasing in the variance of these variables. As shown in Table 9 trend inflation increases the annualized standard deviations of both variables. Again, the increase is sharper in the case of the multi-sector economy. When trend inflation increases from 0% to 4% the standard deviation of consumption increases 6bp in the standard one-sector model versus 45bp in the calibrated multi-sector one. In the case of labor, an increase in trend inflation going from 0% to 4% generates an increase in the standard deviation of 13bp in the standard one-sector model compared to a 54bp increase in the calibrated multi-sector model of Case 5. I conclude that higher trend inflation increases the volatility of consumption and labor for both the one-sector and the multi-sector models, but the effect is proportionally more important for latter case, hence explaining the welfare results. This result is particularly relevant for current dicussions about optimal inflation targeting since it highlights how welfare costs of inflation are larger in multi-sector models even at moderate levels of inflation.

7.2 Monetary Policy

Monetary policy and trend inflation. The volatility results of the previous section may be particularly sensitive to the parameters ϕ_c and ϕ_π used in the Taylor rule given by equation (8). To study how trend inflation affects the volatility of labor and consumption for different combinations of these parameters I perform the following exercise. First, I define a grid for the possible values that parameters ϕ_c and ϕ_π can take. Then for each possible combination (ϕ_c, ϕ_π) of this grid, I solve the system and perform a Montecarlo simulation similar to the one described at the beginning of this section. The resulting combinations of standard deviations (σ_C, σ_L) are illustrated in Figure 13 for trend inflation ranging from 0% to 4%.

A few things are worth noticing in this figure. First, at the lowest level of trend inflation, the clusters of feasible points (σ_C, σ_L) of Cases 1 and 5 overlap. This implies neither the one-sector economy nor the multi-sector economy clearly offer lower feasible combinations (σ_C, σ_L). However, this changes when trend inflation equals 2% since the cluster of points of the multi-sector economy tends to be above that of the one-sector economy. Thus the multi-sector economy offers worse volatility combinations, something that would be reflected in lower welfare. When trend inflation equals 4%, the multi-sector economy is clearly dominated by the one-sector economy; for any given value of σ_L , the feasible value of σ_C is higher in the multi-sector economy of Case 5. Notice also how the cluster of feasible points of Case 5 decreases as trend inflation increases due to the fact that higher trend inflation makes finding a unique equilibrium less likely, everything else constant. I conclude that trend inflation worsens the possible combinations (σ_C, σ_L) that a monetary authority could target, and this phenomenon becomes proportionally more relevant in the multi-sector economy.

To draw the previous conclusion I compared simulations done with different shocks.[10] To isolate the differences that may be caused by having different productivity processes, it is useful to compare Cases 2 and 5, since these cases use the same shock structure for simulations. As seen in the previous sections, Case 2 tends to generate very similar results as the one-sector economy since, other than the productivity shocks, the only difference between those two cases is the number of sectors. Comparing Case 2 and Case 5 in Figure 13 makes it clear how the heterogeneity of Case 5 generates worse volatility combinations and less stable equilibria: the cluster of points of Case 5 is always to the north-east relative to Case 2, and the distance between the remaining points increases as trend inflation increases.

Monetary policy and the business cycle. To understand the challenges associated with monetary policy in the calibrated multi-sector economy Figures 14 and 15 present a comparison of the feasible pairs (σ_C, σ_π) for the grid of policy parameters (ϕ_c, ϕ_π). An exercise like this is relevant for monetary authorities seeking to minimize ad hoc welfare losses of the type

$$\mathscr{L} \;=\; \chi\sigma_\pi^2 + (1-\chi)\,\sigma_c^2, \tag{43}$$

for $\chi \in [0,1]$. Figure 14 shows that the cluster of points in the one-sector economy is not greatly affected by the level of trend inflation, implying that an optimal solution to (43) may not vary much with the level of trend inflation. On the other hand, in the case of the calibrated multi-sector economy, the results are similar to the (σ_C, σ_L) frontier. At lower levels of trend inflation the cluster of feasible volatility combinations in the one-sector model and the calibrated multi-sector model overlap. As trend inflation increases the cluster corresponding to the multi-sector economy shifts right, and finding unique equilibria becomes less likely. Note that this does not happen for the multi-sector economy with homogeneous price stickiness and no production networks (Case 2). Again, the results for this case are similar to the one-sector economy in the sense that trend

[10] As explained in Section 5, the one-sector model uses only 2 shocks: an aggregate monetary shock and an aggregate productivity shock. Although the multi-sector model uses the same monetary shock, it introduces sectoral productivity shocks. These two different shock processes yield similar moments for consumption, labor and inflation under the baseline calibration.

inflation does not alter much the cluster of feasible volatility combinations. A comparison of Cases 2 and 5 highlights the effect of heterogeneity holding the shock structure constant; the heterogeneity of Case 5 worsens the trade-off between inflation and output volatility and makes equilibria less stable.

Finally, to understand what dimensions of heterogeneity drive the latter result, Figure 14 displays the feasible frontiers for the other cases described in Table 3.

Figure 16a highlights how heterogeneity of price stickiness does not greatly affect the level of the feasible volatility combinations but it does affect the existence of equilibria. This comes from comparing Cases 2 and 3 that only differ in the heterogeneity of price stickiness. Case 3 displays clusters of points at similar levels than Case 2, but the cluster of feasible points shrinks as trend inflation increases.

On the other hand Figure 16b emphasizes how production networks amplify the volatility levels. We can draw this conclusion by comparing Case 2 and Case 4, which only differ in having production networks. The cluster of points related to Case 2 offer higher standard deviations of output for each level of standard deviation of inflation. As trend inflation increases both clusters of Cases 2 and 4 move to the right, but the movement is quantitatively more important in the economy with production networks of Case 4.

Alternative Taylor rules. By targeting the aggregate inflation rate, the Taylor rule of equation (8) implicitly weights sectoral inflation rates using consumption shares. A natural question to ask is whether this is the best inflation measure to track. Related to this question, Aoki (2001) argues that in a model with two sectors, one with sticky prices, the other one with flexible prices, the optimal monetary policy is to target sticky-price inflation. The paper argues that stabilizing the inflation rate of the sticky-price sector is sufficient to maintain the relative price at its efficient value. However, in a multi-sector model with heterogeneity of price stickiness the optimal inflation measure becomes less clear. To explore how different inflation measures affect the feasible pairs (σ_C, σ_π) that may be relevant for minimizing an ad hoc loss like in (43) I solve and simulate the model with the following alternative Taylor rule,[11]

[11] This corresponds to the log-linearized version of equation (8).

$$\hat{I}_t = \rho_I \hat{I}_{t-1} + (1-\rho_I)\left[\phi_c \hat{C}_t + \phi_\pi \hat{\Pi}_t^{alt}\right] + z_t^p \tag{44}$$

where

$$\hat{\Pi}_t^{alt} := \sum_{i=1}^{N} \alpha_i^{alt} \hat{\Pi}_{i,t}. \tag{45}$$

The alternative inflation measure $\hat{\Pi}_t^{alt}$ is defined according to (45). In the two-sector case studied in Aoki (2001), $\alpha_{sticky}^{alt} = 1$ and $\alpha_{flexible}^{alt} = 0$, so the inflation target corresponds to the sticky sector.

In this spirit I study two different possibilities. First, I define the weights as a normalized measure of price stickiness, $\alpha_i^{alt} = \frac{\theta_i}{\sum_{i=1}^{N} \theta_i}$. Second, I adjust by the weighted out-degree of each sector, $\alpha_i^{alt} = \frac{\theta_i O_i}{\sum_i^N \theta_i O_i}$. Then, I solve and simulate the model for different combinations of the parameters (ϕ_π, ϕ_c). The results are reported in Figure 16 and Figure 17 for the economy described in Case 5. The cluster of resulting points is then compared to the cluster produced by the standard Taylor rule, i.e. using $\alpha_i^{alt} = \alpha_i$.

The results show that the the cluster of points produced by these types of Taylor rules are largely similar and sometimes offer worse volatility combinations than the standard Taylor rule that targets aggregate inflation. This result is not surprising since the ability of the monetary authority to affect relative prices in the multi-sector model is severely limited by using a monetary tool – the interest rate – that is common to every sector.

8 Conclusions

This paper shows that a disaggregated analysis of a monetary economy captures several important elements that are relevant for monetary policy. Compared to a one-sector model, the relationship between inflation and output is flatter, and different types of sectoral distortions play a role in determining inflation. Sectors with more flexible prices and which are more central in the production network become more important in determining inflation.

This flatter relationship between inflation and output affect the trade-off faced by monetary authorities seeking to minimize inflation and close output-gaps. Monetary policy becomes harder to implement, in the sense that the same set of policy tools yields worse outcomes in terms of variance of inflation, output, and labor. Trend inflation can amplify these effects and make equilibria less stable, something that should be carefully analyzed when studying dynamic responses to different shocks.

Furthermore, the paper analyzes Taylor rules with alternative measures of inflation targets in the multi-sector model. Relative to a standard Taylor rule, the alternative rules are not effective in decreasing the volatility of consumption and inflation. This highlights the difficulty of stabilizing relative price distortions using a monetary tool common to every sector.

The disaggregated analysis of this paper also highlights that trend inflation may be costlier than what an aggregated model without heterogeneity would predict. The welfare losses of trend inflation are amplified relative to an aggregated model, even at moderate levels of trend inflation. Quantitatively this is caused by distortions in relative sectoral prices associated with higher inflation. The relevance of this type of inefficiency in explaining the welfare costs of higher trend inflation highlights the importance of modeling sectoral heterogeneity in economies with frictions.

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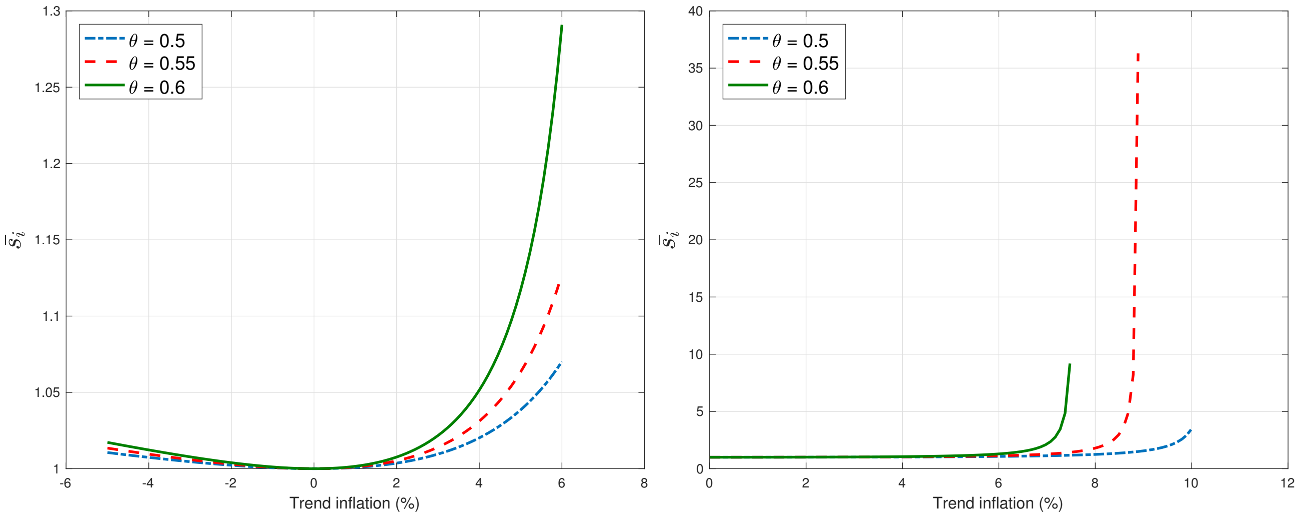
9 Figures and Tables

Figure 1: Steady State Within-Sector Price Dispersion



(a) Trend inflation and minimum price dispersion

(b) Trend inflation and Determinacy

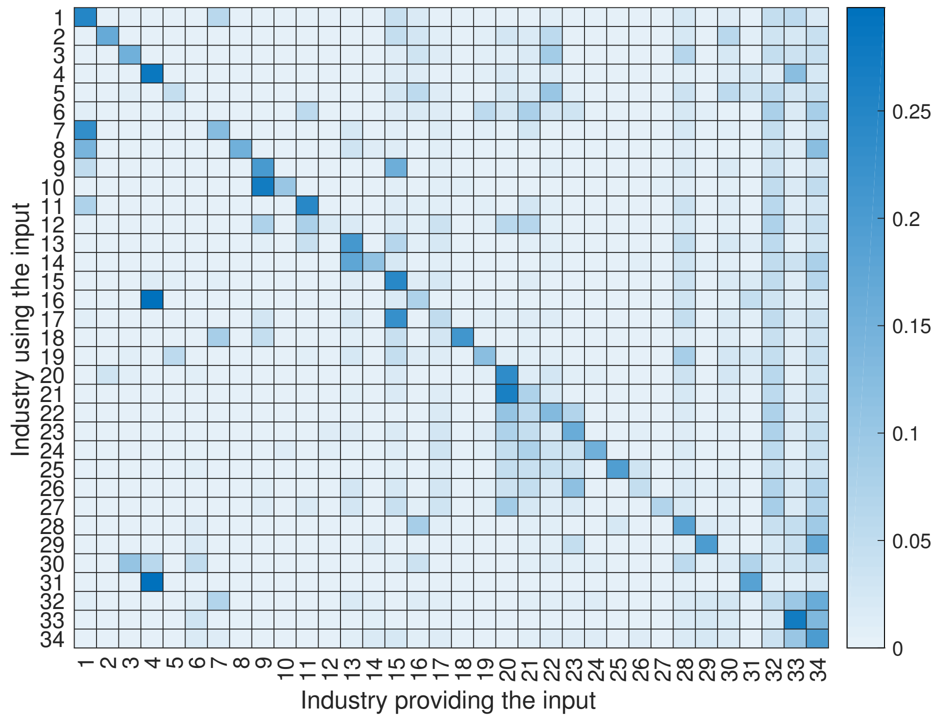
Notes: The figures illustrate the steady state values of the within-sector measure of price dispersion according to equation (29). The elasticity of substitution is fixed at $\epsilon = 7$. Figure 2a shows that $\overline{s}_i$ is minimized when trend inflation is 0%. Figure 2b shows that the determinacy region of $\overline{s}_i$ shrinks for higher values of the price stickiness parameter θ_i.

Table 1: Sectoral Productivity Shocks and Intermediate Inputs

Sector		ρ_i	σ_{ε_i}*100	$1-\delta_i$	O_i
1.	Agriculture	0.72	3.69	0.54	0.79
2.	Metal mining	0.83	6.85	0.48	0.21
3.	Coal mining	0.94	5.70	0.26	0.29
4.	Oil and gas extraction	0.93	5.70	0.27	0.95
5.	Non-metallic mining	0.78	4.28	0.28	0.13
6.	Construction	0.81	1.46	0.55	0.22
7.	Food and kindred products	0.23	1.32	0.69	0.36
8.	Tobacco	0.84	5.57	0.48	0.15
9.	Textile mill products	0.70	1.67	0.63	0.67
10.	Apparel	0.89	1.30	0.64	0.13
11.	Lumber and wood	0.82	2.89	0.61	0.48
12.	Furniture and fixtures	0.83	1.88	0.52	0.05
13.	Paper and allied	0.54	2.31	0.57	0.63
14.	Printing, publishing and allied	0.95	2.29	0.43	0.18
15.	Chemicals	0.75	2.85	0.53	1.16
16.	Petroleum and coal products	0.65	5.99	0.17	0.45
17.	Rubber and misc plastics	0.71	2.01	0.53	0.44
18.	Leather	0.23	2.48	0.60	0.22
19.	Stone, clay, glass	0.91	1.72	0.45	0.31
20.	Primary metal	0.85	2.45	0.63	1.08
21.	Fabricated metal	0.81	1.60	0.54	0.70
22.	Machinery, non-electrical	0.98	3.71	0.56	0.68
23.	Electrical machinery	0.98	3.43	0.51	0.58
24.	Motor vehicles	0.73	2.81	0.77	0.21
25.	Transportation equipment & ordnance	0.39	5.94	0.49	0.22
26.	Instruments	0.92	3.03	0.40	0.13
27.	Misc. manufacturing	0.79	2.77	0.54	0.10
28.	Transportation	0.87	1.87	0.42	1.01
29.	Communications	0.92	2.19	0.42	0.37
30.	Electric utilities	0.96	2.42	0.20	0.49
31.	Gas utilities	0.83	5.55	0.10	0.46
32.	Trade	0.77	1.40	0.36	1.55
33.	Finance Insurance and Real Estate	0.74	1.43	0.33	1.14
34.	Services	0.95	1.44	0.34	1.97

Notes: The table displays the estimated autoregressive coefficients for the sectoral productivity shocks and the standard deviations of the normally distributed shocks according to the methodology explained in Section 5.1. For each industry the table also reports the share of intermediate inputs $1-\delta_i$ and weighted out-degree $O_i := \sum_{i'=1}^{N} \omega_{i',i}$.

Figure 2: Calibrated Production Network



Notes: The figure depicts a heatmap of the calibrated input-output coefficients $\omega_{i,i'}$ according to the methodology presented in Section 5.2 using the $KLEMS$ dataset. The definition of economic sectors corresponds to the one in Table 1.

Table 2: Business Cycle Parameters

Parameter	Description	Value
σ	Constant relative risk aversion	1
φ	Inverse of Frisch elasticity of labor	1
ϵ	Constant elasticity of substitution	7
ϕ_π	Inflationary response of the Taylor Rule	2
ϕ_c	Output-gap response of the Taylor Rule	$\frac{0.5}{4}$
ρ_I	Interest rate smoothing parameter of the Taylor Rule	0.8
ρ_m	Autoregressive coefficient of aggregate monetary shock	0.15
σ_m	Standard deviation of aggregate monetary shock	0.24
ρ_p	Autoregressive coefficient of aggregate productivity shock	0.95
σ_p	Standard deviation of aggregate productivity shock	0.45

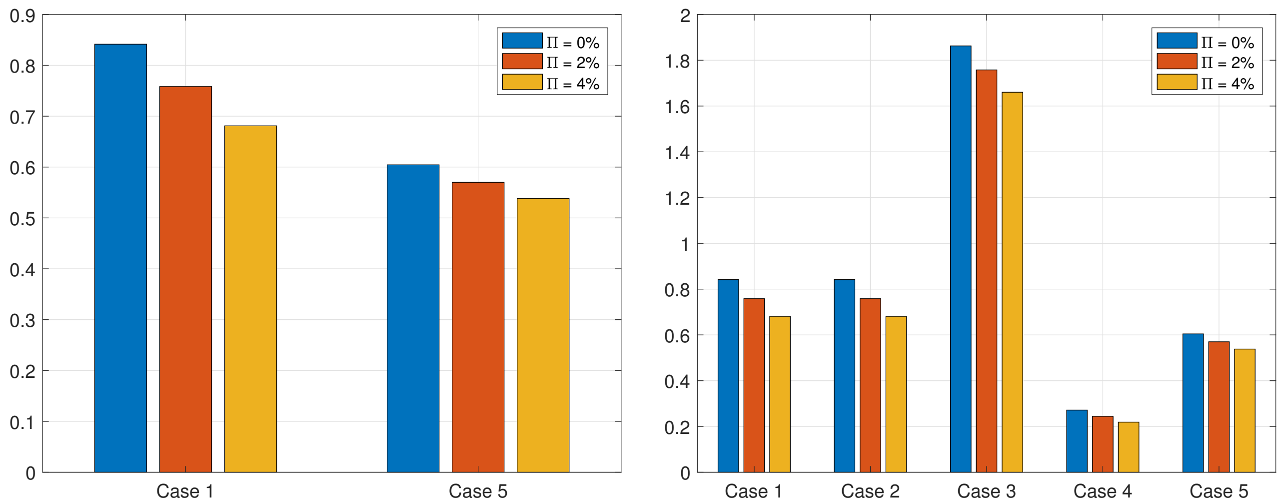
Table 3: Five different economies

Model	Nº of Sectors	Production Network	Frequencies	Share of Intermediate Inputs
1. Standard	1	no network	mean frequency, BLS	0
2. No Network, Homogeneous	34	no network	mean frequency, BLS	0
3. No Network, Heterogeneous	34	no network	calibrated frequencies, BLS	0
4. Network, Homogenous	34	calibrated, $KLEMS$	mean frequency, BLS	calibrated, $KLEMS$
5. Network, Heterogeneous	34	calibrated, $KLEMS$	calibrated frequencies, BLS	calibrated, $KLEMS$

Notes: The table displays the main characteristics of the four different economies studied under the baseline calibration of the model.

Figure 3: Sensitivity of Inflation to the output gap

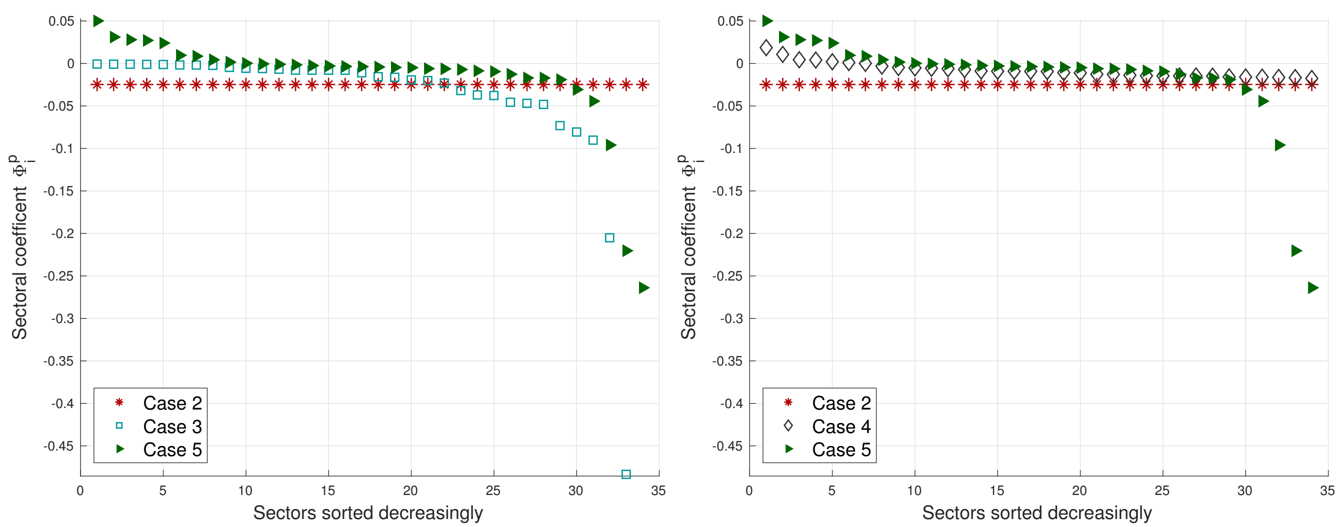
(a) Cases 1 and 5 (b) All Cases



Notes: The figure displays the coefficient Φ_c from equation (39) for 3 different values of trend inflation and for the 5 different economies detailed in Table 3. Case 1 corresponds to the standard one-sector model while Case 5 represents the multi-sector economy. See Table 3 for details about the other Cases.

Figure 4: Sectoral Price Gap Coefficients, Φ_i^p

(a) Effect of heterogeneity of price stickiness (b) Effect of production networks

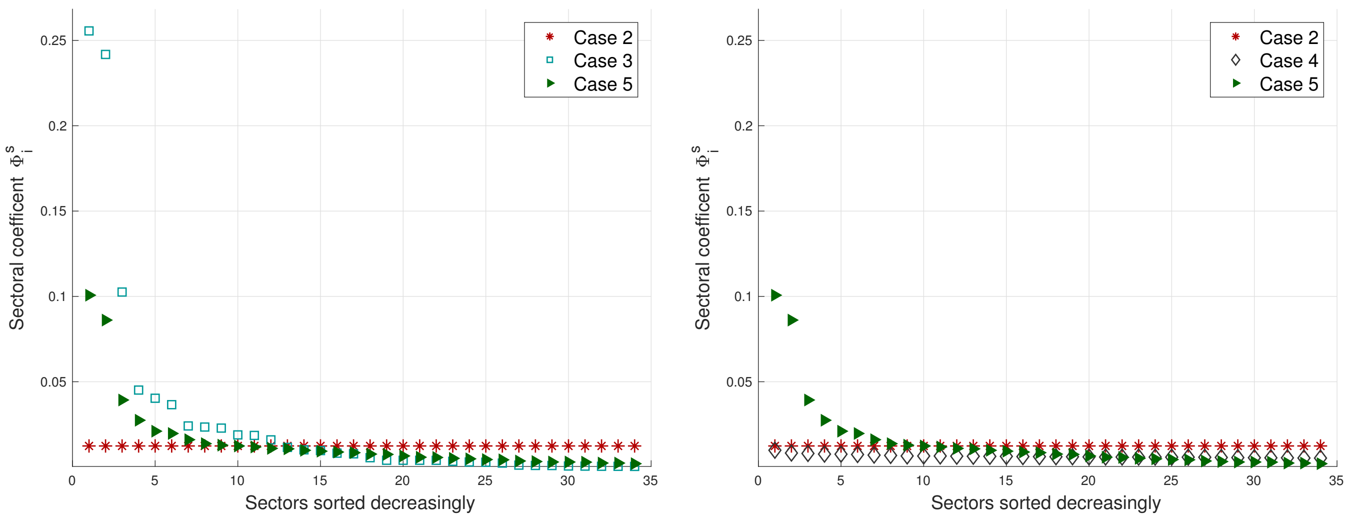


Notes: The figure displays the coefficient Φ_i^p from equation (39) with no trend inflation under the baseline calibration. Each coefficient indicates the partial response of aggregate inflation to sectoral price gaps defined as $\hat{p}_{i,t} := \log\left(\frac{P_{i,t}}{P_t}\right) - \log\left(\frac{P_i}{P}\right)$. Comparing Case 2 and Case 3 in Figure 4a highlights the effect of heterogeneity of price stickiness on the coefficients Φ_i^p. Comparing Case 2 and Case 4 in Figure 4b highlights the effect of production networks on the coefficients Φ_i^p. See Table 3 for a complete definition of each case.

Figure 5: Within-Sector Price Dispersion Gap Coefficients, Φ_i^s

(a) Effect of heterogeneity of price stickiness

(b) Effect of production networks

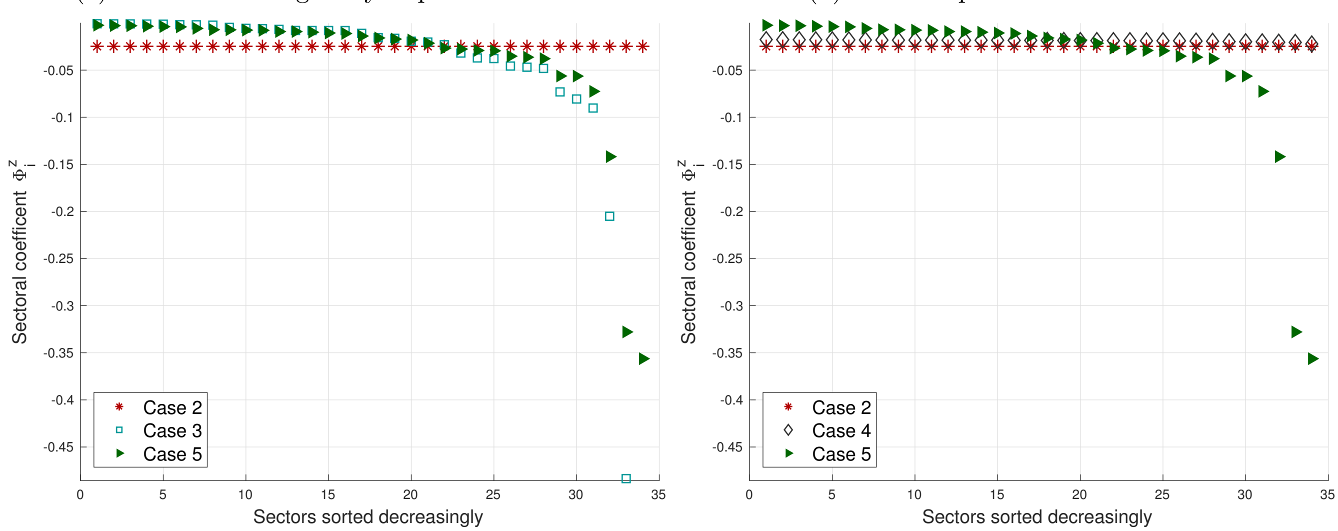


Notes: The figure displays the coefficient Φ_i^p from equation (39) with no trend inflation under the baseline calibration. Comparing Case 2 and Case 3 in Figure 5a highlights the effect of price stickiness heterogeneity on the coefficients Φ_i^p. Comparing Case 2 and Case 4 in Figure 5b highlights the effect of production networks on the coefficients Φ_i^p. See Table 3 for a complete definition of each case.

Figure 6: Productivity Shock Coefficients, Φ_i^z

(a) Effect of heterogeneity of price stickiness

(b) Effect of production networks



Notes: The figure displays the coefficient Φ_i^z from equation (39) with no trend inflation under the baseline calibration. Comparing Case 2 and Case 3 in Figure 6a highlights the effect of price stickiness heterogeneity on the coefficients Φ_i^z. Comparing Case 2 and Case 4 in Figure 6b highlights the effect of production networks on the coefficients Φ_i^z. See Table 3 for a complete definition of each case.

Table 4: Correlation Coefficients-Characteristics

		Sectoral Coefficient					
		(a)	(b)	(c)	(d)	(e)	(f)
Economy	Key characteristic	Φ_i^p	$\lvert\Phi_i^p\rvert$	Φ_i^s	$\lvert\Phi_i^s\rvert$	Φ_i^z	$\lvert\Phi_i^z\rvert$
Case 3	Different stickiness, θ_i	0.75	-0.75	-0.75	-0.75	0.75	-0.75
Case 4	Different centrality, O_i	0.99	-0.49	0.98	0.98	-0.98	0.98

Notes: The table reports the correlations between different sectoral coefficients and two characteristics highlighted by Cases 3 and 4: heterogeneity of price stickiness, measured as θ_i , and heterogeneity of sectoral centrality, measured as the weighted out-degree O_i. In some cases the coefficients can take different signs, so the table also reports the correlation between the absolute value of the coefficient and the key characteristics. The coefficients are computed under no trend inflation. Details about Cases 3 and 4 are described in Table 3.

Table 5: Sectoral NKPC Coefficients with No Trend Inflation

	Sector	Φ_i^p	Φ_i^s	Φ_i^z	Φ_i^π	Φ_i^ψ
1.	Agriculture	-0.23	0.10	-0.36	0.03	0.00
2.	Metal mining	-0.27	0.09	-0.33	0.03	0.00
3.	Coal mining	-0.01	0.01	-0.02	0.03	0.00
4.	Oil and gas extraction	0.03	0.01	-0.04	0.03	0.00
5.	Non-metallic mining	-0.02	0.01	-0.02	0.03	0.00
6.	Construction	-0.02	0.01	-0.03	0.03	0.00
7.	Food and kindred products	-0.02	0.02	-0.03	0.03	0.00
8.	Tobacco	-0.02	0.01	-0.01	0.03	0.00
9.	Textile mill products	-0.01	0.01	-0.01	0.03	0.00
10.	Apparel	-0.01	0.00	-0.00	0.03	0.00
11.	Lumber and wood	-0.02	0.01	-0.02	0.03	0.00
12.	Furniture and fixtures	-0.01	0.00	-0.00	0.03	0.00
13.	Paper and allied	-0.01	0.01	-0.01	0.03	0.00
14.	Printing, publishing and allied	-0.00	0.00	-0.00	0.03	0.00
15.	Chemicals	0.03	0.01	-0.02	0.03	0.00
16.	Petroleum and coal products	-0.01	0.01	-0.04	0.03	0.00
17.	Rubber and misc plastics	-0.00	0.00	-0.01	0.03	0.00
18.	Leather	-0.01	0.00	-0.00	0.03	0.00
19.	Stone, clay, glass	-0.00	0.00	-0.01	0.03	0.00
20.	Primary metal	-0.03	0.03	-0.07	0.03	0.00
21.	Fabricated metal	0.01	0.01	-0.01	0.03	0.00
22.	Machinery, non-electrical	0.02	0.00	-0.01	0.03	0.00
23.	Electrical machinery	-0.01	0.01	-0.01	0.03	0.00
24.	Motor vehicles	-0.05	0.02	-0.06	0.03	0.00
25.	Transportation equipment & ordnance	-0.01	0.00	-0.01	0.03	0.00
26.	Instruments	-0.01	0.00	-0.00	0.03	0.00
27.	Misc. manufacturing	-0.01	0.00	-0.00	0.03	0.00
28.	Transportation	0.01	0.01	-0.03	0.03	0.00
29.	Communications	-0.00	0.00	-0.01	0.03	0.00
30.	Electric utilities	-0.01	0.01	-0.04	0.03	0.00
31.	Gas utilities	-0.10	0.04	-0.14	0.03	0.00
32.	Trade	0.00	0.02	-0.06	0.03	0.00
33.	Finance Insurance and Real Estate	0.03	0.01	-0.03	0.03	0.00
34.	Services	0.05	0.01	-0.01	0.03	0.00
	Minimum	-0.27	0.00	-0.36	0.03	0.00
	Maximum	0.05	0.10	-0.00	0.03	0.00
	Mean	-0.02	0.01	-0.04	0.03	0.00
	Sum	-0.60	0.49	-1.42	1.00	0.00

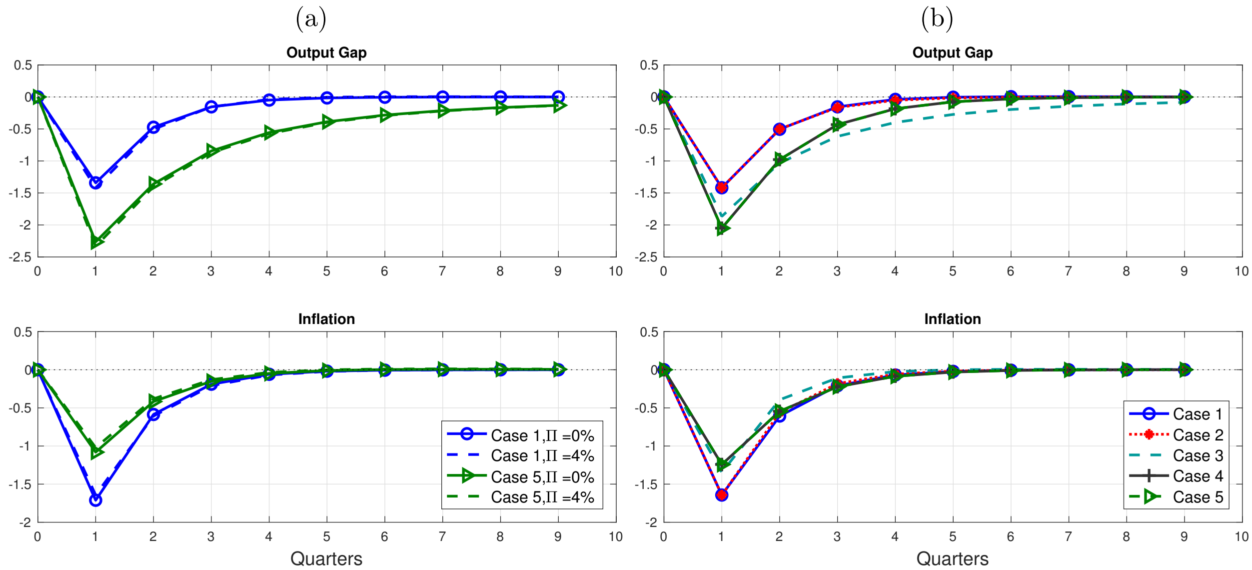
Notes: The table reports the calibrated parameters of the generalized NKPC in (39), under no trend inflation. Results correspond to Case 5 described in Table 3.

Table 6: Sectoral NKPC Coefficients with Trend Inflation of 2%

Sector		Φ_i^p	Φ_i^s	Φ_i^z	Φ_i^π	$\Phi_i^\psi * 10^{-3}$
1.	Agriculture	-0.21	0.10	-0.34	0.03	0.13
2.	Metal mining	-0.25	0.08	-0.32	0.03	0.13
3.	Coal mining	-0.01	0.01	-0.01	0.03	0.06
4.	Oil and gas extraction	0.03	0.01	-0.04	0.03	0.08
5.	Non-metallic mining	-0.02	0.01	-0.02	0.03	0.06
6.	Construction	-0.02	0.01	-0.03	0.03	0.08
7.	Food and kindred products	-0.01	0.02	-0.03	0.03	0.06
8.	Tobacco	-0.02	0.01	-0.01	0.03	0.04
9.	Textile mill products	-0.01	0.01	-0.01	0.03	0.04
10.	Apparel	-0.00	0.00	-0.00	0.03	0.01
11.	Lumber and wood	-0.01	0.01	-0.02	0.03	0.06
12.	Furniture and fixtures	-0.00	0.00	-0.00	0.03	0.02
13.	Paper and allied	-0.00	0.01	-0.01	0.03	0.05
14.	Printing, publishing and allied	-0.00	0.00	-0.00	0.03	0.01
15.	Chemicals	0.03	0.01	-0.02	0.03	0.06
16.	Petroleum and coal products	-0.01	0.01	-0.03	0.03	0.08
17.	Rubber and misc plastics	0.00	0.00	-0.01	0.03	0.04
18.	Leather	-0.00	0.00	-0.00	0.03	0.01
19.	Stone, clay, glass	-0.00	0.00	-0.01	0.03	0.04
20.	Primary metal	-0.03	0.03	-0.07	0.03	0.10
21.	Fabricated metal	0.01	0.01	-0.01	0.03	0.03
22.	Machinery, non-electrical	0.02	0.00	-0.01	0.03	0.02
23.	Electrical machinery	0.00	0.00	-0.01	0.03	0.04
24.	Motor vehicles	-0.04	0.02	-0.05	0.03	0.10
25.	Transportation equipment & ordnance	-0.01	0.00	-0.01	0.03	0.04
26.	Instruments	-0.00	0.00	-0.00	0.03	0.01
27.	Misc. manufacturing	-0.00	0.00	-0.00	0.03	0.02
28.	Transportation	0.01	0.01	-0.03	0.03	0.08
29.	Communications	-0.00	0.00	-0.01	0.03	0.04
30.	Electric utilities	-0.00	0.01	-0.03	0.03	0.08
31.	Gas utilities	-0.09	0.04	-0.14	0.03	0.12
32.	Trade	0.00	0.02	-0.05	0.03	0.09
33.	Finance Insurance and Real Estate	0.03	0.01	-0.02	0.03	0.07
34.	Services	0.05	0.01	-0.01	0.03	0.02
	Minimum	-0.25	0.00	-0.34	0.03	0.01
	Maximum	0.05	0.10	-0.00	0.03	0.13
	Mean	-0.02	0.01	-0.04	0.03	0.06
	Sum	-0.57	0.46	-1.34	1.01	1.92

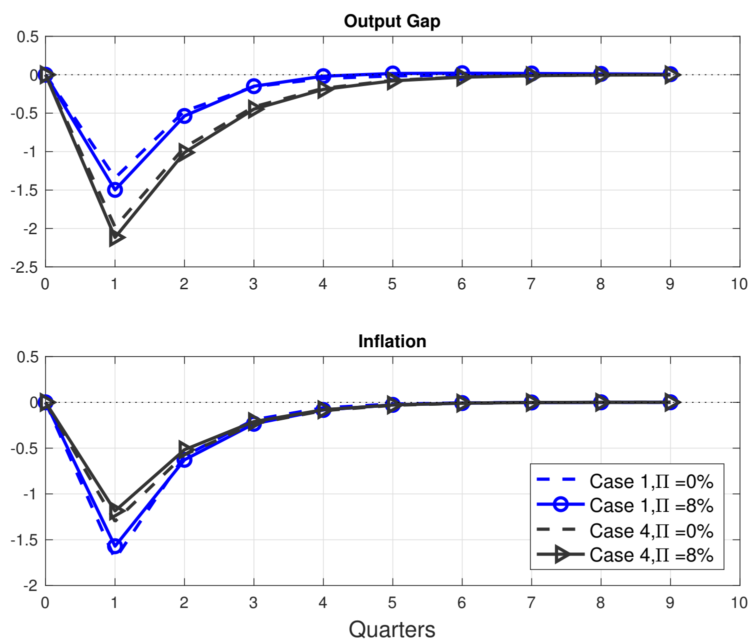
Notes: The table reports the calibrated parameters of the generalized NKPC in (39), under trend inflation equal to 2% annually. Results correspond to Case 5 described in Table 3.

Figure 7: Impulse Response Functions to a 1% Monetary Shock



Notes: Figure 7a depicts the annualized IRF to a 1% monetary shock for Cases 1 and 4 under the baseline calibration and two different levels of trend inflation. Figure 7b shows the annualized IRF to a 1% monetary shock for all cases described in Table 3 under the baseline calibration and trend inflation equal to 4%.

Figure 8: Impulse Response Functions to a 1% Monetary Shock, All Cases



Notes: The figure depicts the annualized IRF to a 1% monetary shock under the baseline calibration and 2 levels of trend inflation. Cases 1 to 4 are described in Table 3.

Table 7: Cumulative response to a 1% monetary shock, baseline calibration

(a) Output gap

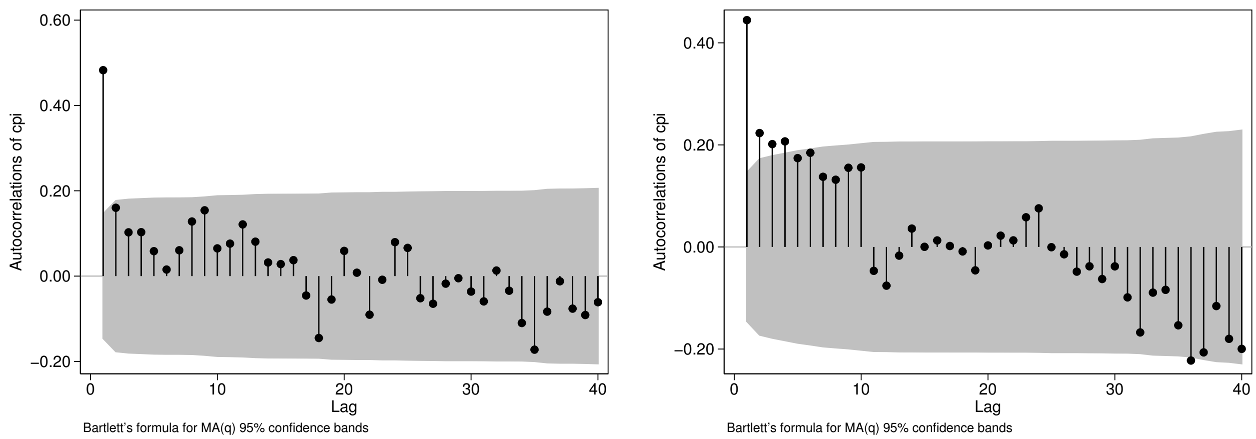
	At impact					At 4th quarter				
	Inflation Trend					Inflation Trend				
Case	0%	2%	4%	6%	8%	0%	2%	4%	6%	8%
1	-0.34	-0.35	-0.35	-0.36	-0.37	-0.50	-0.52	-0.53	-0.54	-0.55
2	-0.34	-0.35	-0.35	-0.36	-0.37	-0.50	-0.52	-0.53	-0.55	-0.56
3	-0.43	-0.45	-0.47	–	–	-0.90	-0.94	-0.98	–	–
4	-0.50	-0.50	-0.51	-0.52	-0.53	-0.88	-0.90	-0.91	-0.93	-0.94
5	-0.57	-0.58	-0.58	–	–	-1.26	-1.28	-1.30	–	–

	At impact					At 4th quarter				
	Inflation Trend					Inflation Trend				
Case	0%	2%	4%	6%	8%	0%	2%	4%	6%	8%
1	-0.43	-0.42	-0.41	-0.40	-0.39	-0.64	-0.63	-0.63	-0.63	-0.63
2	-0.43	-0.42	-0.41	-0.40	-0.39	-0.64	-0.62	-0.61	-0.60	-0.58
3	-0.36	-0.35	-0.34	–	–	-0.52	-0.49	-0.47	–	–
4	-0.32	-0.32	-0.31	-0.30	-0.30	-0.55	-0.54	-0.52	-0.51	-0.50
5	-0.27	-0.26	-0.26	–	–	-0.42	-0.41	-0.39	–	–

(b) Inflation gap

Notes: The tables report the cumulative annualized response of the output gap and inflation gap to a 1% monetary shock under the baseline calibration for different levels of trend inflation and for the 5 cases described in Table 3. In Cases 3 and 5, a unique equilibrium does not exist for trend inflation above 4%, so the values are not reported. The output gap is defined as the log-difference between aggregate value-added output relative to its steady state level. The inflation gap is defined as the log-difference between gross aggregate inflation and its steady state level.

Figure 9: Inflation Autocorrelation



(a) Case 1

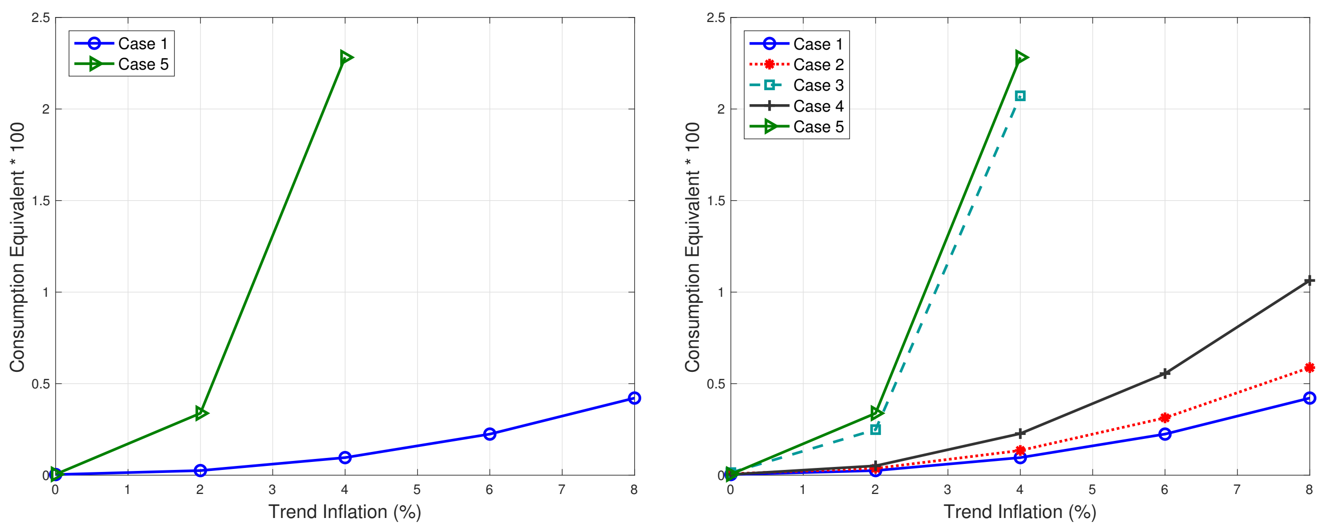
(b) Case 5

Notes: The figure depicts the autocorrelation coefficients of aggregate inflation for Cases 1 and 5 described in Table 3. The autocorrelations are computed by solving each model and performing a Montecarlo simulation with monetary and productivity shocks. See Section 7 for details about the simulation.

Figure 10: Consumption Equivalent

(a) Cases 1 and 5

(b) All Cases



Notes: For each level of trend inflation the figures show welfare costs measured as the consumption equivalent implicitly defined by equation (42). See Table 3 for the definition of Cases 1 to 5. For trend inflation above 4% a unique equilibrium does not exist for Cases 3 and 5.

Table 8: Consumption Equivalent

(a) Baseline Calibration

Case	Trend Inflation 0%	2%	4%	6%	8%
1	0.00	0.03	0.10	0.22	0.42
2	0.01	0.04	0.14	0.31	0.59
3	0.01	0.25	2.07	-	-
4	0.00	0.05	0.23	0.55	1.06
5	0.00	0.34	2.28	-	-

(b) Share of intermediate inputs = 0.65

Case	Trend Inflation 0%	2%	4%	6%	8%
1	0.00	0.03	0.10	0.22	0.42
2	0.01	0.04	0.14	0.31	0.59
3	0.01	0.25	2.07	-	-
4	0.00	0.05	0.27	0.66	1.28
5	0.00	0.38	2.68	-	-

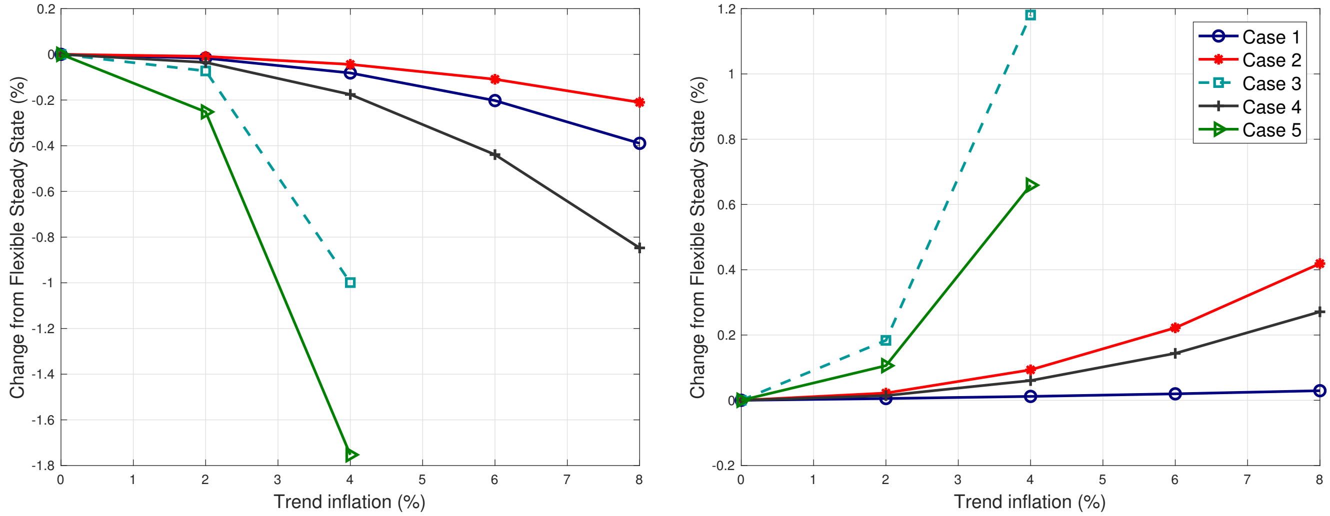
Case	Trend Inflation 0%	2%	4%	6%	8%
1	0.00	0.03	0.10	0.22	0.42
2	0.01	0.04	0.14	0.31	0.59
3	0.02	0.25	2.07	-	-
4	0.01	0.05	0.23	0.56	1.07
5	0.01	0.34	2.28	-	-

(c) Only Sectoral Shocks

Case	Trend Inflation 0%	2%	4%	6%	8%
1	0.01	0.04	0.15	0.36	0.71
2	0.01	0.05	0.21	0.51	1.02
3	0.02	0.44	–	–	–
4	0.01	0.08	0.37	0.93	1.86
5	0.01	0.64	–	–	–

(d) Higher elasticity of substitution, $\epsilon = 10$

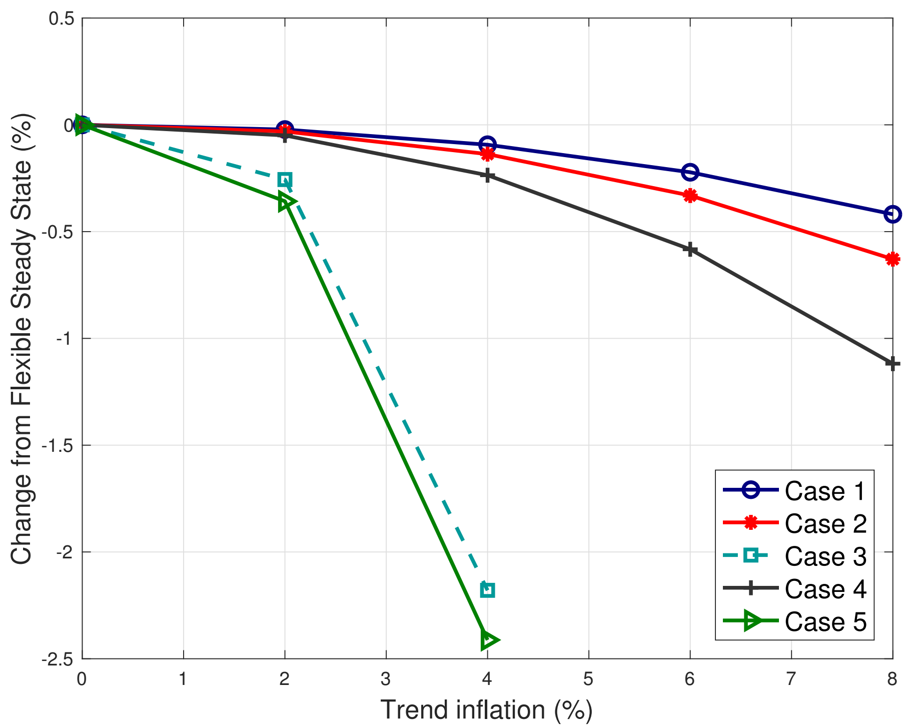
Figure 11: Steady State Values



(a) Aggregate Consumption (b) Aggregate Labor

Notes: For each level of trend inflation the figures show the difference between the steady state level of a variable in the sticky price equilibrium relative to the flexible price equilibrium. The definition of Cases 1 to 5 is detailed in Table 3.

Figure 12: Steady State TFP



Notes: For each level of trend inflation the figure shows the difference between steady state TFP in the sticky price equilibrium relative to the flexible price equilibrium. The definition of Cases 1 to 5 is detailed in Table 3.

Table 9: Annualized Standard Deviations

(a) Aggregate Consumption

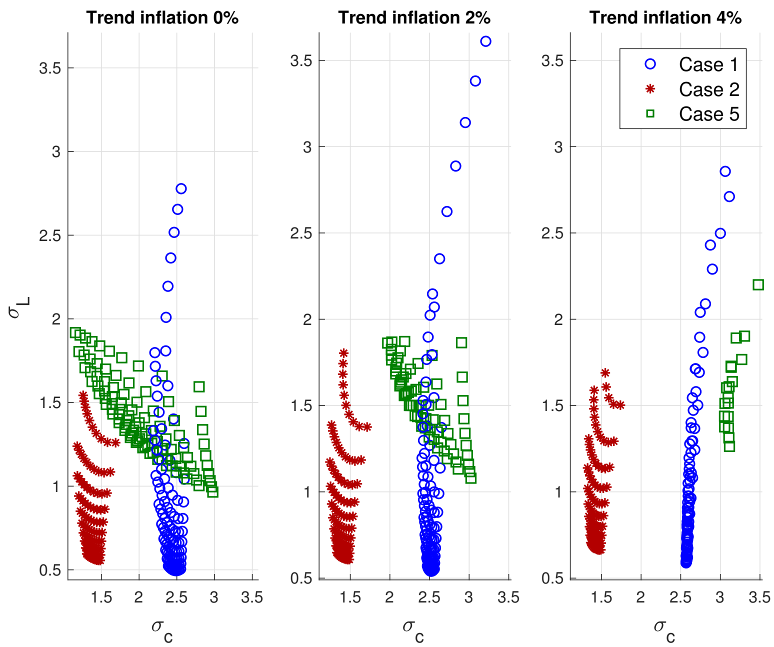
	Trend Inflation				
Case	0%	2%	4%	6%	8%
1.	2.54	2.57	2.60	2.64	2.69
2.	1.49	1.50	1.52	1.53	1.55
3.	1.57	1.62	1.70	–	–
4.	2.78	2.81	2.85	2.89	2.96
5.	2.68	2.77	3.13	–	–

	Trend Inflation				
Case	0%	2%	4%	6%	8%
1.	0.71	0.77	0.84	0.92	1.01
2.	0.77	0.84	0.91	0.99	1.09
3.	1.15	1.31	1.56	–	–
4.	0.93	1.01	1.09	1.18	1.28
5.	1.20	1.37	1.74	–	–

(b) Aggregate Labor

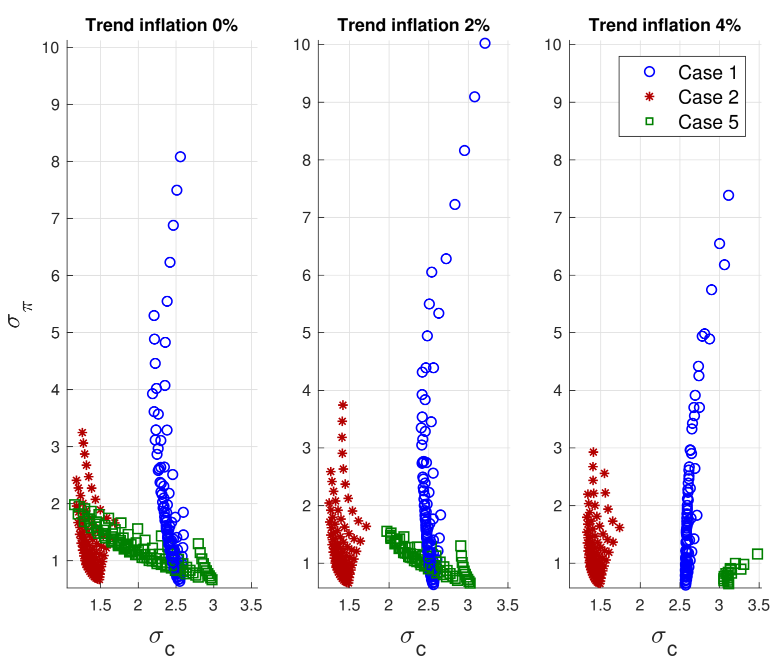
Notes: For each case described in Table 3, these tables report the annualized standard deviations of consumption and labor under different values of trend inflation.

Figure 13: Feasible (σ_C, σ_L) Frontiers



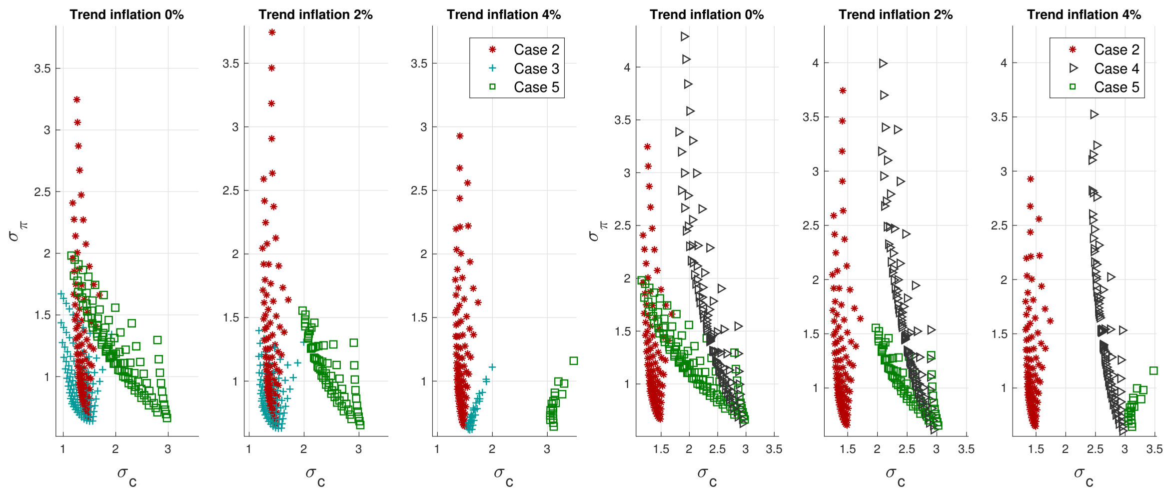
Notes: The figure displays pairs of annualized standard deviations (σ_c, σ_L) for different combinations of policy parameters (ϕ_c, ϕ_π) of the Taylor rule. The parameter ϕ_π takes values in $[1.1, 3]$ while ϕ_c takes values in $[0.05, 1]$. For each combination of parameters the solution of the model is computed and a Montecarlo simulation is performed to calculate the variances of labor and consumption. The definition of Cases 1, 2 and 5 is detailed in Table 3.

Figure 14: Feasible (σ_C, σ_π) Frontiers



Notes: The figure displays pairs of annualized standard deviations (σ_c, σ_π) for different combinations of policy parameters (ϕ_c, ϕ_π) of the Taylor rule. The parameter ϕ_π takes values in $[1.1, 3]$ while ϕ_c takes values in $[0.05, 1]$. For each combination of parameters the solution of the model is computed and a Montecarlo simulation is performed to calculate the variances of consumption and inflation. The definition of Cases 1, 2 and 5 is detailed in Table 3.

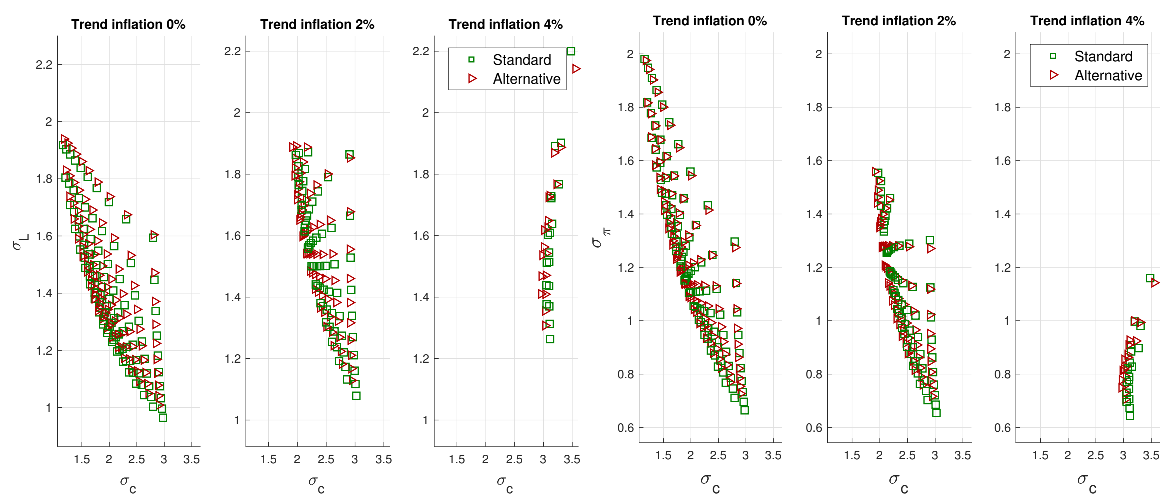
Figure 15: Feasible (σ_C, σ_π) Frontiers



(a) The effect of heterogeneity of price stickiness (b) The effect of production networks

Notes: The figure displays pairs of annualized standard deviations (σ_c, σ_π) for different combinations of policy parameters (ϕ_c, ϕ_π) of the Taylor rule. The parameter ϕ_π takes values in $[1.1, 3]$ while ϕ_c takes values in $[0.05, 1]$. For each combination of parameters the solution of the model is computed and a Montecarlo simulation is performed to calculate the variances of consumption and inflation. The definition of Cases 1, 2 and 5 is detailed in Table 3.

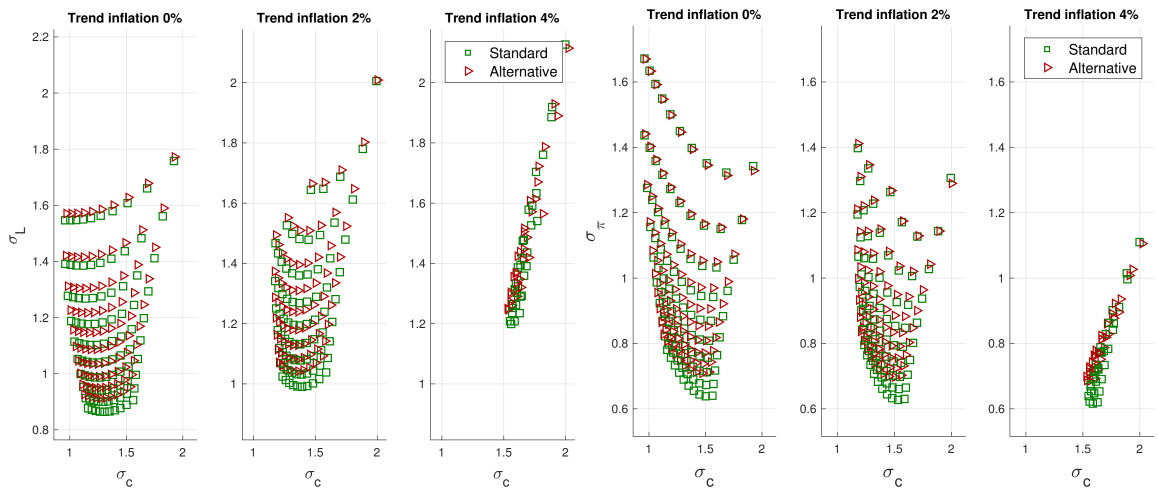
Figure 16: Alternative Taylor Rules, Sectors Weighted by Stickiness



(a) Feasible combinations (σ_c, σ_L) (b) Feasible combinations (σ_c, σ_π)

Notes: The figure displays pairs of annualized standard deviations (σ_c, σ_π) for different combinations of policy parameters (ϕ_c, ϕ_π) of the Taylor rule. The parameter ϕ_π takes values in $[1.1, 3]$ while ϕ_c takes values in $[0.05, 1]$. The Standard case corresponds to the standard Taylor rule. The Alternative case uses sectoral weights defined as $\alpha_i^{alt} = \theta_i / \sum_{i'=1}^{N} \theta_{i'}$. See section 7.2 for details. For each combination of parameters the solution of the model is computed and a Montecarlo simulation is performed to calculate the variances of consumption and inflation. The results are reported for case 5 described in Table 3.

Figure 17: Alternative Taylor Rules, Sectors Weighted by Stickiness and Centrality



(a) Feasible combinations (σ_c, σ_L) (b) Feasible combinations (σ_c, σ_π)

Notes: The figure displays pairs of annualized standard deviations (σ_c, σ_π) for different combinations of policy parameters (ϕ_c, ϕ_π) of the Taylor rule. The parameter ϕ_π takes values in $[1.1, 3]$ while ϕ_c takes values in $[0.05, 1]$. The Standard case corresponds to the standard Taylor rule. The Alternative case uses sectoral weights defined as $\alpha_i^{alt} = \theta_i O_i / \sum_{i'=1}^{N} (\theta_{i'} O_{i'})$. See section 7.2 for details. For each combination of parameters the solution of the model is computed and a Montecarlo simulation is performed to calculate the variances of consumption and inflation. The results are reported for case 5 described in Table 3.

A Appendix to Section 3

A.1 Firms

A.1.1 The marginal cost

The total cost of producing quantity $Y_{(i,j),t}$ is given by

$$\mathcal{C}\left(Y_{(i,j),t}\right) = \min_{L_{(i,j),t},\left\{X_{(i,j),(i',j'),t}\right\}} W_{i,t}L_{(i,j),t} + \sum_{i'=1}^{N}\int_0^1 X_{(i,j),(i',j'),t}P_{(i',j'),t}dj'$$

subject to the production function (9) and the CES aggregator (11). The CES structure of the sectoral bundles allows to simplify the latter problem and write it in terms of just sectoral variables conditional on some optimality conditions. Given a certain level for the sectoral bundle from a sector i', $X_{(i,j),i'}$, a firm (i,j) faces the following cost minimization problem

$$\begin{aligned} \min_{\left\{X_{(i,j),(i',j'),t}\right\}} & \quad \int_0^1 X_{(i,j),(i',j'),t}P_{(i',j'),t}dj' \\ \text{s.t.} & \\ X_{(i,j),i',t} &= \left(\int_0^1 X_{(i,j),(i',j'),t}^{\frac{\epsilon-1}{\epsilon}}dj'\right)^{\frac{\epsilon}{\epsilon-1}}. \end{aligned} \tag{46}$$

The solution to this problem gives the demand of firm (i,j) for intermediate goods produced by firm (i',j') conditional on (i,j)'s demand of the sectoral bundle $X_{(i,j),i',t}$,

$$X_{(i,j),(i',j')} = \left(\frac{P_{(i',j'),t}}{P_{i',t}}\right)^{-\epsilon} X_{(i,j),i',t}, \tag{47}$$

where the sectoral price index is defined by $P_{i',t} = \left(\int_0^1 P_{(i',j'),t}^{1-\epsilon}dj'\right)^{\frac{1}{1-\epsilon}}$. Given this optimality condition, the problem of cost minimization can be written in terms of sectoral variables:

$$\mathcal{C}\left(Y_{(i,j),t}\right) = \min_{L_{(i,j),t},\left\{X_{(i,j),i',t}\right\}} W_{i,t}L_{(i,j),t} + \sum_{i'=1}^{N} X_{(i,j),i',t}P_{i',t}dj'$$

subject to the production function (9).

Given a quantity of output $Y_{(i,j),t}$, optimality implies the following conditional demands for inputs

$$L_{(i,j),t} = \delta \frac{MC_{i,t}}{W_{i,t}} Y_{(i,j),t}, \tag{48}$$

$$X_{(i,j),i',t} = \omega_{i,i'} \frac{MC_{i,t}}{P_{i',t}} Y_{(i,j),t}, \tag{49}$$

where $MC_{i,t}$ is the nominal marginal cost of production in industry i, given by

$$MC_{i,t} = \frac{P_{i,t}^{m} \left(\frac{W_{i,t}}{\delta} \right)^{\delta}}{Z_{i,t}^{p}}, \tag{50}$$

and where $P_{i,t}^{m}$ is the industry specific price index of materials, defined by

$$P_{i,t}^{m} = \prod_{i'=1}^{N} \left(\frac{P_{i',t}}{\omega_{i,i'}} \right)^{\omega_{i,i'}}.$$

Note that the marginal cost is the same for every firm within sector i, independently of the price charged. This is an implication of the CRS assumption. Also, this assumption implies that the total cost of production is simply

$$\mathcal{C}\left(Y_{(i,j),t}\right) = MC_{i,t} Y_{(i,j),t}. \tag{51}$$

A.1.2 Firm's total demand

Since a firm (i,j) sells its output as a consumption good to the representative household and as an intermediate good to all the firms from (possibly) all sector of the economy, its total demand is given by

$$Y_{(i,j),t} = C_{(i,j),t} + \sum_{i'=1}^{N} \int_0^1 X_{(i',j'),(i,j),t} dj'.$$

Given the CES structure of both the consumption and production bundle then $C_{(i,j),t} = \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} C_{i,t}$ and $X_{(i',j'),(i,j)} = \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} X_{(i',j'),i,t}$. As a result we can replace these expression into the total demand of firm (i,j) to get

$$Y_{(i,j),t} \quad = \quad \left(\frac{P_{(i,j),t}}{P_{i,t}}\right)^{-\epsilon} \left[C_{i,t} + \sum_{i'=1}^{N} X_{i',i,t}\right].$$

where $X_{i',i,t} := \int_0^1 X_{(i',j'),i,t} dj'$ is the total demand of sector i' for inputs from sector i. Defining $Y_{i,t} := C_{i,t} + \sum_{i'=1}^{N} X_{i',i,t}$ yields equation (17).

A.1.3 Optimal pricing condition

The solution to the price-setting problem (19) is

$$p^*_{i,t} \quad = \quad \frac{\epsilon}{\epsilon-1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \Pi^{\epsilon}_{i,t,t+k} Y_{i,t+k} mc_{i,t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \frac{\Pi^{\epsilon}_{i,t,t+k}}{\Pi_{t,t+k}} Y_{i,t+k}}, \tag{52}$$

where $\Pi_{i,t,t+k} := \frac{P_{i,t+k}}{P_{i,t}}$ is gross sectoral inflation between periods t and $t+k$ in industry i, $mc_{i,t+k} := MC_{i,t+k}/P_{t+k}$ is the real marginal cost in this industry and $\Pi_{t,t+k}$ is aggregate gross inflation. The optimal pricing condition (52) can be written in terms of the auxiliary variables $\psi_{i,t}$ and $\Delta_{i,t}$ as

$$p^*_{i,t} \quad = \quad \frac{\epsilon}{\epsilon-1} \frac{\psi_{i,t}}{\Delta_{i,t}} \tag{53}$$

where

$$\psi_{i,t} \quad := \quad \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \theta^k \Pi^{\epsilon}_{i,t,t+k} Y_{i,t+k} mc_{i,t+k}$$

and

$$\Delta_{i,t} \quad := \quad \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \theta^k \frac{\Pi^{\epsilon}_{i,t,t+k}}{\Pi_{t,t+k}} Y_{i,t+k}.$$

We can use the recursive structure of $\psi_{i,t}$ and $\Delta_{i,t}$ to write these terms as

$$\psi_{i,t} = Y_{i,t} C_t^{-\sigma} mc_{i,t} + \theta_i \beta \mathbb{E}_t \left[\Pi_{i,t+1}^{\epsilon} \psi_{i,t+1} \right], \tag{54}$$

$$\Delta_{i,t} = Y_{i,t} C_t^{-\sigma} + \theta_i \beta \mathbb{E}_t \left[\frac{\Pi_{i,t+1}^{\epsilon}}{\Pi_{t+1}} \Delta_{i,t+1} \right], \tag{55}$$

where $\Pi_{i,t+1}$ and Π_{t+1} denote gross sectoral and aggregate inflation between periods t and $t+1$, respectively.

Given the time-dependent structure of prices, at each moment of time there is a measure $1-\theta_i$ of firms re-optimizing their price and a measure θ_i keeping their price from the previous period. Thus the sectoral price satisfies

$$P_{i,t} = \left(\theta_i P_{i,t-1}^{1-\epsilon} + (1-\theta_i) \left(P_{i,t}^{*} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

$$1 = \left(\theta_i \left(\frac{P_{i,t-1}}{P_{i,t}} \right)^{1-\epsilon} + (1-\theta_i) \left(\frac{P_{i,t}^{*}}{P_t} \frac{P_t}{P_{i,t}} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow p_{i,t}^{*} = \left(\frac{1-\theta_i \Pi_{i,t}^{\epsilon-1}}{1-\theta_i} \right)^{\frac{1}{1-\epsilon}} p_{i,t}. \tag{56}$$

Where lowercase letters denote prices relative to the aggregate price index.

On the other hand, we can get rid of $\Delta_{i,t}$ by combining equations (56) and (20). These equations imply

$$\Delta_{i,t} = \frac{\epsilon}{\epsilon-1} \left(\frac{1-\theta_i \Pi_{i,t}^{\epsilon-1}}{1-\theta_i} \right)^{\frac{1}{\epsilon-1}} \frac{1}{p_{i,t}} \psi_{i,t},$$

using this expression back in (55) we get equation (22) from the text.

B Appendix to section 4

B.1 Equilibrium system

The equilibrium is defined by the system of equations below. Equations (57) to (60) are necessary for every sector $i = 1, .., N$, where $mc_{i,t}$ is determined by (26) conditional on the other endogenous variables. Equation (61) is a definition needed to keep track of the evolution of the relative prices given the inflation rates. The Euler equation (62) and the Taylor equation (63) are the aggregate equations needed to close the model.Together, the system defines $5N + 2$ equations determining $5N$ sectoral endogenous variables, $\{\psi_{i,t}, s_{i,t}, \Pi_{i,t}, p_{i,t}, Y_{i,t}\}_{i=1}^{N}$, plus value-added output, i.e. C_t, and the nominal interest rate, I_t.

$$\psi_{i,t} = Y_{i,t} C_t^{-\sigma} mc_{i,t} + \theta_i \beta \mathbb{E}_t \left[\Pi_{i,t+1}^{\epsilon} \psi_{i,t+1} \right], \tag{57}$$

$$\left(\frac{1 - \theta_i \Pi_{i,t}^{\epsilon-1}}{1 - \theta_i} \right)^{\frac{1}{\epsilon-1}} \psi_{i,t} = \frac{\epsilon - 1}{\epsilon} Y_{i,t} C_t^{-\sigma} p_{i,t} + \theta_i \beta \mathbb{E}_t \left[\Pi_{i,t+1}^{\epsilon-1} \left(\frac{1 - \theta_i \Pi_{i,t+1}^{\epsilon-1}}{1 - \theta_i} \right)^{\frac{1}{\epsilon-1}} \psi_{i,t+1} \right], \tag{58}$$

$$s_{i,t} = (1 - \theta_i) \left(\frac{1 - \theta_i \Pi_{i,t}^{\epsilon-1}}{1 - \theta_i} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta_i \Pi_{i,t}^{\epsilon} s_{i,t-1}, \tag{59}$$

$$Y_{i,t} = \alpha_i \frac{C_t}{p_{i,t}} + \sum_{i'=1}^{N} \omega_{i',i} \frac{mc_{i',t}}{p_{i,t}} s_{i',t} Y_{i',t}, \tag{60}$$

$$p_{i,t} = p_{i,t-1} \frac{\Pi_{i,t}}{\Pi_t}, \tag{61}$$

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{I_t}{\Pi_{t+1}} \right], \tag{62}$$

$$\frac{I_t}{\overline{I}} = \left(\frac{I_t}{\overline{I}} \right)^{\rho_I} \left[\left(\frac{C_t}{\overline{C}} \right)^{\phi_c} \left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\phi_\pi} \right]^{1-\rho_I} Z_t^m. \tag{63}$$

B.2 Log-linearized Equilibrium System

The online appendix shows how a log-linearization of the model around a positive inflation steady-state equilibrium is characterized by the following system of equations,

$$0 = \mathbb{E}_t\left[\hat{C}_{t+1}\right] + \frac{1}{\sigma}\left(\mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] - \hat{I}_t\right) - \hat{C}_t \tag{64}$$

$$0 = \rho_I \hat{I}_{t-1} + (1-\rho_I)\left(\phi_c \hat{C}_t + \phi_\pi \hat{\Pi}_t\right) + z_t^m - \hat{I}_t \tag{65}$$

$$\mathbf{0} = \text{diag}\left(\overline{\Pi}_i^\epsilon \theta_i\right)\hat{\mathbf{s}}_{t-1} + \text{diag}\left(\overline{\Pi}_i^{\epsilon-1}\theta_i\epsilon \frac{\overline{\Pi}_i - 1}{1-\overline{\Pi}_i^{\epsilon-1}\theta_i}\right)\hat{\mathbf{\Pi}}_t - \hat{\mathbf{s}}_t \tag{66}$$

$$\begin{aligned}\mathbf{0} = {} & (\mathbf{1}\sigma - \mathbb{Y}_c)\hat{C}_t - \text{diag}\left(\frac{\theta_i}{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}\overline{\Pi}_i^{\epsilon-1}\right)\hat{\mathbf{\Pi}}_t - (\mathbf{I}+\mathbb{Y}_p)\hat{\mathbf{p}}_t - \mathbb{Y}_s\hat{\mathbf{s}}_t + \text{diag}\left(\frac{1}{\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}\right)\hat{\boldsymbol{\psi}}_t - \mathbb{Y}_z\hat{\mathbf{z}}_t^p \\ & - \text{diag}\left(\frac{\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\left(\epsilon - 1 - \frac{\theta_i\overline{\Pi}_i^{\epsilon-1}}{1-\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)}{\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}\right)\mathbb{E}_t\left[\hat{\mathbf{\Pi}}_{t+1}\right] - \text{diag}\left(\frac{\theta_i\beta\overline{\Pi}_i^{\epsilon-1}}{\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}\right)\mathbb{E}_t\left[\hat{\boldsymbol{\psi}}_{t+1}\right]\end{aligned} \tag{67}$$

$$\begin{aligned}\mathbf{0} = {} & \frac{1+\varphi}{1+(1-\delta)\varphi}\left(\mathbb{Y}_c - (1-\delta)\sigma\mathbf{1}\right)\hat{C}_t + \frac{1+\varphi}{1+(1-\delta)\varphi}\left(\mathbf{\Omega}+\mathbb{Y}_p\right)\hat{\mathbf{p}}_t + \frac{1}{1+(1-\delta)\varphi}\left((1+\varphi)\mathbb{Y}_s + \delta\varphi\mathbf{I}\right)\hat{\mathbf{s}}_t - \text{diag}\left(\frac{1}{1-\theta_i\beta\Pi_i^\epsilon}\right)\hat{\boldsymbol{\psi}}_t \\ & + \frac{1+\varphi}{1+(1-\delta)\varphi}\left(\mathbb{Y}_z - \mathbf{I}\right)\hat{\mathbf{z}}_t^p + \text{diag}\left(\frac{\theta_i\beta\epsilon\overline{\Pi}_i^\epsilon}{\left(1-\theta_i\beta\overline{\Pi}_i^\epsilon\right)}\right)\mathbb{E}_t\left[\hat{\mathbf{\Pi}}_{t+1}\right] + \text{diag}\left(\frac{\theta_i\beta\overline{\Pi}_i^\epsilon}{1-\theta_i\beta\Pi_i^\epsilon}\right)\mathbb{E}_t\left[\hat{\boldsymbol{\psi}}_{t+1}\right]\end{aligned} \tag{68}$$

$$\mathbf{0} = \hat{\mathbf{p}}_{t-1} + \left(\mathbf{I} - \mathbf{1}\boldsymbol{\alpha}'\right)\hat{\mathbf{\Pi}}_t - \hat{\mathbf{p}}_t. \tag{69}$$

Where bold letters of signs denote vectors, diag (a_i) denotes a $N\times N$ diagonal matrix with elements a_i in the i-th diagonal, $\mathbb{Y}_c$ is a $N\times 1$ vector and $\mathbb{Y}_s, \mathbb{Y}_p, \mathbb{Y}_z$ are $N\times N$ matrices determined in steady state.

B.3 Aggregate Production Function

B.3.1 Domar weights

We can manipulate the sectoral market clearing condition for goods (60) to get the following equation for determining the Domar weights,

$$\eta_{i,t} = \alpha_i + \sum_{i'=1}^{N}\omega_{i',i}\frac{mc_{i',t}s_{i',t}}{p_{i',t}}\eta_{i',t},$$

where $\eta_{i,t} := \frac{Y_{i,t}p_{i,t}}{C_t}$. By redefining $\tilde{\omega}_{i,i',t} = \omega_{i,i'}\frac{mc_{i,t}s_{i,t}}{p_{i,t}}$ the Domar weights can be determined as a function of other variables,

$$\boldsymbol{\eta}_t = \left(\mathbf{I} - \tilde{\mathbf{\Omega}}_t'\right)^{-1}\boldsymbol{\alpha}. \tag{70}$$

On the other hand, inputs hired by an individual firm can be written in terms of total inputs hired by that firm's sector by computing the ratio $\kappa_{(i,j),t} := L_{(i,j),t}/L_{i,t}$. Using the individual labor demand (14) we get the ratio is equal to $\kappa_{(i,j),t} = \left(\frac{P_{i,t}}{P_{(i,j),t}}\right)^{\epsilon} s_{i,t}^{-1}$. The structure of the problem implies the same ratio for intermediate inputs from other industries, i.e. $\kappa_{(i,j),t} = X_{(i,j),i',t}/X_{i,i',t} = \left(\frac{P_{i,t}}{P_{(i,j),t}}\right)^{\epsilon} s_{i,t}^{-1}$. Using this result, sectoral production can be written in terms of sectoral inputs utilized by firms within that sector,

$$\begin{aligned} Y_{i,t} &= \int_0^1 Z_{i,t}^p \left(\kappa_{(i,j),t} L_{i,t}\right)^{\delta} \prod_{i'=1}^{N} \left(\kappa_{(i,j),t} X_{i,i',t}\right)^{\omega_{i,i'}} dj \\ &= Z_{i,t}^p L_{i,t}^{\delta} \prod_{i'=1}^{N} X_{i,i',t}^{\omega_{i,i'}} \int_0^1 \kappa_{(i,j),t} dj \\ &= Z_{i,t}^p L_{i,t}^{\delta} \prod_{i'=1}^{N} X_{i,i',t}^{\omega_{i,i'}}. \end{aligned} \tag{71}$$

Next, we need to express sectoral labor demand as a function of aggregate labor. We do this by computing the ratio of sectoral labor demand to total labor demand, $\kappa_{i,t} := L_{i,t}/L_t$. Using equation(24), we find the ratio

$$\kappa_{i,t} = \frac{\left(\frac{mc_{i,t}}{w_{i,t}} \frac{1}{p_{i,t}} s_{i,t}\right) \eta_{i,t}}{\left[\sum_{i'=1}^{N} \left(\frac{mc_{i',t}}{w_{i',t}} \frac{1}{p_{i',t}} s_{i',t} \eta_{i',t}\right)^{1+\varphi}\right]^{\frac{1}{1+\varphi}}}, \tag{72}$$

which is another function of the endogenous variables of the model.

Finally, using (25) we can write intermediate input demands as functions of prices, price dispersion and Domar weights,

$$\begin{aligned} X_{i,i',t} &= \omega_{i,i'} \frac{mc_{i,t} s_{i,t}}{p_{i,t}} \frac{Y_{i,t} p_{i,t}}{C_t} \frac{C_t}{Y_{i',t} p_{i',t}} Y_{i',t} \\ &= \omega_{i,i'} \frac{mc_{i,t} s_{i,t}}{p_{i,t}} \frac{\eta_{i,t}}{\eta_{i',t}} Y_{i',t}. \end{aligned} \tag{73}$$

B.3.2 Proof of Proposition

Proof. Combining (71), (73) and the definition of $\kappa_{i,t}$, the log of sectoral output is given by

$$\log\left(Y_{i,t}\right) \;=\; a_{i,t}+z_{i,t}^{p}+\delta\log\left(L_{t}\right)+\sum_{i'=1}^{N}\omega_{i,i'}\log\left(Y_{i',t}\right),$$

where $a_{i,t} := \delta\log\left(\kappa_{i,t}\right)+\sum_{i'=1}^{N}\omega_{i,i'}\log\left(\omega_{i,i'}\frac{mc_{i,t}s_{i,t}}{p_{i,t}}\frac{\eta_{i,t}}{\eta_{i',t}}\right)$ is an endogenous variable of the model. Expressing this equation in vectorial form, solving for the sectoral vector of total output and using the constant returns to scale assumption of the production functions yields

$$\log\left(\mathbf{Y}_{t}\right) \;=\; \left(\mathbf{I}-\boldsymbol{\Omega}\right)^{-1}\left(\mathbf{a}_{t}+\mathbf{z}_{t}^{p}\right)+\log\left(L_{t}\right). \tag{74}$$

To finally get the aggregate production function let us recall from the definition of Domar weight that $\eta_{i,t}=\frac{Y_{i,t}p_{i,t}}{C_{t}}$. On the other hand, from the optimality condition of the household $\frac{\alpha_{i}}{C_{i,t}}=\frac{p_{i,t}}{C_{t}}$, thus the Domar weight can be expressed in terms of sectoral consumption as $\eta_{i,t}=Y_{i,t}\frac{\alpha_{i}}{C_{i,t}}$, or

$$\log\left(C_{i,t}\right) \;=\; \tilde{\alpha}_{i,t}+\log\left(Y_{i,t}\right), \tag{75}$$

where $\tilde{\alpha}_{i,t} := \log\left(\frac{\alpha_{i}}{\eta_{i,t}}\right)$. Since $\log\left(C_{t}\right)=\sum_{i=1}^{N}\alpha_{i}\log\left(C_{i,t}\right)$ this represents the final element needed to compute the aggregate production function. Combining (74) and (75) the aggregate production function is

$$C_{t} \;=\; TFP_{t}\cdot L_{t}$$

$$TFP_{t} \;=\; \exp\left\{\boldsymbol{\alpha}'\left[\tilde{\boldsymbol{\alpha}}_{t}+\left(\mathbf{I}-\boldsymbol{\Omega}\right)^{-1}\left(\mathbf{a}_{t}+\mathbf{z}_{t}^{p}\right)\right]\right\}.$$

Evaluating this expression in steady state yields equation (33).

□

C Appendix to Section 6

C.1 Generalized Aggregate New Keynesian Philips Curve

By manipulating equation (68) we get an expression for $\hat{\boldsymbol{\psi}}_t$ that can be substituted in (67) to get

$$
\begin{aligned}
\hat{\mathbf{\Pi}}_t \;=\;& \left[\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbf{1}\sigma-\mathbb{Y}_c)+\frac{1+\varphi}{1+(1-\delta)\varphi}\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbb{Y}_c-(1-\delta)\sigma\mathbf{1})\right]\hat{C}_t \\
&+\left[\frac{1+\varphi}{1+(1-\delta)\varphi}\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbf{\Omega}+\mathbb{Y}_p)-\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbf{I}+\mathbb{Y}_p)\right]\hat{\mathbf{p}}_t \\
&+\left[\frac{1}{1+(1-\delta)\varphi}\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)((1+\varphi)\mathbb{Y}_s+\delta\varphi\mathbf{I})-\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)\mathbb{Y}_s\right]\hat{\mathbf{s}}_t \\
&+\operatorname{diag}\left(\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1+\epsilon(\Pi_i-1)+\frac{\theta_i\overline{\Pi}_i^{\epsilon-1}}{1-\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)\right)\beta\mathbb{E}_t\left[\hat{\mathbf{\Pi}}_{t+1}\right] \\
&+\operatorname{diag}\left(\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(\overline{\Pi}_i-1\right)\right)\beta\mathbb{E}_t\left[\hat{\boldsymbol{\psi}}_{t+1}\right] \\
&+\left[\frac{1+\varphi}{1+(1-\delta)\varphi}\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbb{Y}_z-\mathbf{I})-\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)\mathbb{Y}_z\right]\hat{\mathbf{z}}_t^p. \qquad (76)
\end{aligned}
$$

The aggregate NKPC is generated by premultiplying equation (76) by the consumption shares $\boldsymbol{\alpha}'$,

$$
\hat{\Pi}_t \;=\; \Phi_c\hat{C}_t+\sum_{i=1}^{N}\left[\Phi_i^p\hat{p}_{i,t}+\Phi_i^s\hat{s}_{i,t}+\beta\Phi_i^\pi\mathbb{E}_t\left[\hat{\Pi}_{i,t+1}\right]+\beta\Phi_i^\psi\mathbb{E}_t\left[\hat{\psi}_{i,t+1}\right]+\Phi_i^z z_{i,t}^p\right] \qquad (77)
$$

where the coefficient of output gap is given by

$$
\Phi_c \;=\; \boldsymbol{\alpha}'\left[\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbf{1}\sigma-\mathbb{Y}_c)+\frac{1+\varphi}{1+(1-\delta)\varphi}\operatorname{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbb{Y}_c-(1-\delta)\sigma\mathbf{1})\right],
$$

and the weights Φ_i^x for $x=p,s,\pi,\psi,z$ represent the i-th element of the $1\times N$ vectors $\mathbf{\Phi}^x$ defined below

$$\mathbf{\Phi}^{p} = \boldsymbol{\alpha}'\left[\frac{1+\varphi}{1+(1-\delta)\varphi}\text{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\Pi_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbf{\Omega}+\mathbb{Y}_p)-\text{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbf{I}+\mathbb{Y}_p)\right]$$

$$\mathbf{\Phi}^{s} = \boldsymbol{\alpha}'\left[\frac{1}{1+(1-\delta)\varphi}\text{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\Pi_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)((1+\varphi)\mathbb{Y}_s+\delta\varphi\mathbf{I})-\text{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)\mathbb{Y}_s\right]$$

$$\mathbf{\Phi}^{\pi} = \boldsymbol{\alpha}'\text{diag}\left(\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1+\epsilon(\Pi_i-1)+\frac{\theta_i\overline{\Pi}_i^{\epsilon-1}}{1-\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)\right)$$

$$\mathbf{\Phi}^{\psi} = \boldsymbol{\alpha}'\text{diag}\left(\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)(\overline{\Pi}_i-1)\right)$$

$$\mathbf{\Phi}^{z} = \boldsymbol{\alpha}'\left[\frac{1+\varphi}{1+(1-\delta)\varphi}\text{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\Pi_i^{\epsilon}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)(\mathbb{Y}_z-\mathbf{I})-\text{diag}\left(\frac{\left(1-\theta_i\overline{\Pi}_i^{\epsilon-1}\right)\left(1-\theta_i\beta\overline{\Pi}_i^{\epsilon-1}\right)}{\theta_i\overline{\Pi}_i^{\epsilon-1}}\right)\mathbb{Y}_z\right].$$

C.1.1 Pasten, Shoenle and Weber 2016

Pasten et al. (2016) study the real effects of multi-sector model with production networks and sectoral heterogeneity of price stickiness. To reproduce this setting I need to set trend inflation equal to zero in every sector, i.e. $\overline{\Pi}_i = 1$. This produces the following NKPC

$$\hat{\Pi}_t = \Phi_c\hat{C}_t + \beta\mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] + \sum_{i=1}^{N}\left[\Phi_i^p\hat{p}_{i,t} + \Phi_i^z z_{i,t}^p\right] \tag{78}$$

where the coefficient of output gap is given by

$$\Phi_c = \boldsymbol{\alpha}'\left[\text{diag}\left(\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right)(\mathbf{1}\sigma-\mathbb{Y}_c)+\frac{1+\varphi}{1+(1-\delta)\varphi}\text{diag}\left(\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right)(\mathbb{Y}_c-(1-\delta)\sigma\mathbf{1})\right],$$

and the weights Φ_i^x for $x = p, z$ represent the i-th element of the $1\times N$ vectors $\mathbf{\Phi}^x$ defined below

$$\mathbf{\Phi}^{p} = \boldsymbol{\alpha}'\left[\frac{1+\varphi}{1+(1-\delta)\varphi}\text{diag}\left(\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right)(\mathbf{\Omega}+\mathbb{Y}_p)-\text{diag}\left(\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right)(\mathbf{I}+\mathbb{Y}_p)\right]$$

$$\mathbf{\Phi}^{z} = \boldsymbol{\alpha}'\left[\frac{1+\varphi}{1+(1-\delta)\varphi}\text{diag}\left(\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right)(\mathbb{Y}_z-\mathbf{I})-\text{diag}\left(\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right)\mathbb{Y}_z\right].$$

The NKPC (78) combines equations (67) and (68) from the log-linearized system. On the other hand, replacing $\overline{\Pi}_i = 1$ in (66) implies the sectoral variables $\hat{s}_{i,t}$ are no longer relevant in the system. Therefore in this case the log-linearized system is simpler and the optimal dynamics can be described with equation (78) and 3 more equations,

$$0 = \mathbb{E}_t\left[\hat{C}_{t+1}\right] + \frac{1}{\sigma}\left(\mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] - \hat{I}_t\right) - \hat{C}_t \tag{79}$$

$$0 = \rho_I \hat{I}_{t-1} + (1-\rho_I)\left(\phi_c \hat{C}_t + \phi_\pi \hat{\Pi}_t\right) + z_t^m - \hat{I}_t \tag{80}$$

$$\mathbf{0} = \hat{\mathbf{p}}_{t-1} + (\mathbf{I} - \mathbf{1}\boldsymbol{\alpha}')\,\hat{\boldsymbol{\Pi}}_t - \hat{\mathbf{p}}_t. \tag{81}$$

C.1.2 Carvalho 2006

The results from Carvalho (2006) can be reproduced by assuming no production networks and no trend inflation. An online appendix shows that $\boldsymbol{\Omega} = \mathbf{0}$ implies $\mathbb{Y}_c = \mathbf{1}$, $\mathbb{Y}_p = -\mathbf{I}$ $\mathbb{Y}_s = \mathbf{0}$ $\mathbb{Y}_z = \mathbf{0}$ and the no trend inflation assumption implies $\overline{\Pi}_i = 1$ for every sector i, so

$$\hat{\Pi}_t = \beta\mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] + \Phi_c\hat{C}_t + \sum_{i=1}^{N}\Phi_i^p \hat{p}_{i,t} + \sum_{i=1}^{N}\Phi_i^z z_{i,t}^p \tag{82}$$

where the coefficients of the NKPC are given by

$$\begin{aligned}
\Phi_c &= (\sigma+\varphi)\sum_{i=1}^{N}\alpha_i\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i} \\
\Phi_i^p &= -(1+\varphi)\left(\alpha_i\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\right) \\
\Phi_i^z &= -(1+\varphi)\frac{(1-\theta_i)(1-\theta_i\beta)}{\theta_i}\alpha_i.
\end{aligned}$$

C.1.3 Ascari and Sbordone 2014

To reproduce the results from Ascari and Sbordone (2014) we need to set $N = 1$, so $\boldsymbol{\Omega} = 0$ $\mathbb{Y}_c = 1, \mathbb{Y}_p = -1, \mathbb{Y}_s = 0, \mathbb{Y}_z = 0$ are now scalars. The NKPC now becomes

$$\hat{\Pi}_t = \Phi^c\hat{C}_t + \Phi^\pi\beta\mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] + \Phi^\psi\beta\mathbb{E}_t\left[\hat{\psi}_{t+1}\right] + \Phi^s\hat{s}_t + \Phi^z z_t^p \tag{83}$$

where the coefficients are now given by

$$\begin{aligned}
\Phi^c &= \left(\frac{\left(1-\theta\overline{\Pi}^{\epsilon-1}\right)\left(1-\theta\beta\Pi^{\epsilon}\right)}{\theta\overline{\Pi}^{\epsilon-1}}\right)(\sigma+\varphi)+\beta\left(1-\theta\overline{\Pi}^{\epsilon-1}\right)\left(1-\overline{\Pi}\right)(1-\sigma) \\
\Phi^\pi &= \left(1+\epsilon\left(1-\theta\overline{\Pi}^{\epsilon-1}\right)(\Pi-1)\right) \\
\Phi^\psi &= \left(1-\theta\overline{\Pi}^{\epsilon-1}\right)\left(\overline{\Pi}-1\right) \\
\Phi^s &= \varphi\left(\frac{\left(1-\theta\overline{\Pi}^{\epsilon-1}\right)\left(1-\theta\beta\overline{\Pi}^{\epsilon}\right)}{\theta\overline{\Pi}^{\epsilon-1}}\right) \\
\Phi^z &= -(1+\varphi)\frac{\left(1-\theta\overline{\Pi}^{\epsilon-1}\right)\left(1-\theta\beta\Pi^{\epsilon}\right)}{\theta\overline{\Pi}^{\epsilon-1}}
\end{aligned}$$

C.1.4 Standard New Keynesian Philips Curve

The standard NKPC can be found by setting trend inflation equal to zero, i.e. $\overline{\Pi} = 1$ in equation (83). This yields

$$\hat{\Pi}_t = \Phi^c\hat{C}_t + \beta\mathbb{E}_t\left[\hat{\Pi}_{t+1}\right] + \Phi^z z_t^p \tag{84}$$

where the coefficients are now given by

$$\begin{aligned}
\Phi^c &= \left(\frac{(1-\theta)(1-\theta\beta)}{\theta}\right)(\sigma+\varphi) \\
\Phi^z &= -(1+\varphi)\frac{(1-\theta)(1-\theta\beta)}{\theta}.
\end{aligned}$$

The term involving $\hat{s}_t$ is ignored in this case since it does not matter in a first order approximation.