TAX SHARING IN INSURANCE MARKETS: A USEFUL PARAMETERIZATION

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ABSTRACT. We use a Principal-Agent framework to evaluate the economic impacts of imposing a tax on insurance payment in presence of moral hazard using a Gamma conditional distribution of losses. Our results show that any tax paid by the insured would the lower his effort to prevent loss, hence increasing insurance payments and decreasing profits. This result is reinforced as the insured becomes more risk averse unless the distribution of losses is uniform. We find that any decrease in the insurer's tax share would generate an overall decrease in welfare unless the insured characteristics prevent him from reacting to the policy.

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1. Introduction

During the past few years, much attention has been placed on possible revenue sources to pay for health care. The recent political interest has been to tax insurance companies: the Baucus plan levies a non-deductible excise tax of 40% on insurance companies. In many insurance markets, the insurance provider determines the premium based on a coverage amount chosen by the insured. As a result, a tax on the premium amount corresponds to a proportional tax on the coverage amount. For example, in the long term care insurance market, the future insured is prompted to choose a monthly benefit during the contract negociation. In this paper, we analyze the economic impacts of imposing a tax on payments out to the insured. The question then is: should the insurance provider pay the full amount of the tax as is proposed or should the insured bear a portion of it? And in case the insured bears a portion of it, should this portion be attributed uniformly across contract types? To answer these questions, we generalize the analysis to a case where the tax could be shared between the insurer and the insured.

Unfortunately, the presence of asymmetric information in insurance markets complicates the analysis (Chiappori and Salanie, 1997, 2000; Dionne et al., 2000, 2012; Villeneuve, 2000; Abbring et al., 2003 to name a few). The lack of care enforcement on the part of the insurer, i.e. moral hazard, complicates the welfare outcome of any policy aiming at redistributing health care coverage (see for example Dionne et al., 1997; Doherty and Smetters, 2005, for empirical evidence of moral hazard in insurance markets, and Ketsche, 2004 for an empirical analysis of the impact of a subsidy on welfare). Indeed, the more health insurance an individual acquires, the lower his risk. This improves the social welfare of those with the greater coverage. But as his coverage increases the insured subsequently invests less in self-protection and consequently increases his use of health care services. This increased demand causes increased use of services that results in higher prices, thus having the opposing effect on social welfare. Because risk-averse consumers would not purchase this additional care if they had to pay the full cost, the value of the extra-service to consumers falls short of the social cost of producing that care. Therefore, although risk sharing increases social well-being, the change in moral hazard induces welfare loss.

Early theoretical studies on moral hazard in insurance markets (Arrow, 1963; Pauly, 1968; Shavell, 1979) proposed solutions to the moral hazard issue. The solutions included (i) an incomplete coverage against loss, which gives the individual an incentive to prevent loss by exposing him to some financial risk (see Wang et al, 2008 and Chiappori et al, 1997; Koc, 2011 for empirical evidence of this solution in the market for automobile insurance and for physician services insurance respectively) and (ii) the observation of care, to link it to the premium or coverage in the event of a claim. The impossibility to fully observe care, however, has led to an increasing literature on the design of optimal contracts (Lewis and Sappington, 1995; Winter, 1992, 2000; Gollier, 2000; Doherty and Smetters, 2005).

In this paper we use a framework allowing for a distribution of losses with care reducing both the probability of a loss and the size of a loss (MasColell, Whinsten and Green, 1995) .[1] In our model, an Agent insures with a Principal. Both have property rights to an uncertain income stream that represents a possible loss from current wealth. The random income stream depends on care or effort on the part of the Agent, to be taken in the future. The Principal establishes a sharing rule on how to share the random income stream. The Principal-Agent relationship involves moral hazard, because the Agent's effort to avoid any loss is unobservable by the Principal, while it ultimately affects the expected profit. Therefore the Principal wants to use the contract to induce the Agent to exert optimal effort to invest in self-protection and/or loss reduction.

It is well known, however, that the Principal-Agent problem (Laffont and Martimort, 2002) is difficult to solve when effort is a continuous variable. Its tractability depends on the ability to simplify an infinite number of global incentive constraints corresponding to an infinite number of possible effort levels and replace them by a local incentive constraint to induce the maximum effort from the agent. This last condition states that the agent is indifferent between choosing a

[1]Economic models of moral hazard in insurance markets differ on their assumptions about the impact of greater care on the distribution of losses faced by the insured (Winter, 2000). When the loss is assumed to be random, the literature distinguishes the effects of moral hazard on two types of expenditures on the part of the insured (Ehrlich and Becker, 1972). The first type of expenditures, called self-protection, reduces the probability of an accident. It refers to an increase in the probability of a zero loss, with no change in the conditional distribution. The second type, called loss reduction, refers to a first-order stochastic reduction with no change in the probability of a loss. In these two cases, the optimal insurance contract is characterized by a premium and a coverage amount that may depend on the loss amount.We use a third approach allowing for an arbitrary distribution of losses with care reducing both the probability of a loss and the size of a loss

given level of effort and increasing it by a slight amount when he receives the risk premium. This so-called 'first order approach' has been one of the most debated issues in contract theory. Mirrlees (1975) has shown that the problem may sometimes have no optimal solution in the class of unbounded sharing rules. Rogerson (1985) shows that the 'original' first-order approach (with an infinite number of global incentive constraints) gives the same solutions as the first order approach when the following two properties are satisfied. The first, the Maximum Likelihood Ratio Property (MLRP), ensures that the Agent is rewarded in the state of nature that is most informative in that he has exerted positive effort. The second property is the Convexity of the Distribution Function Condition (CDFC) which ensures that higher profit (of the Principal) is a signal of higher effort on the part of the Agent. The problem is that the CDFC property is very restrictive and tremendously limits the list of possible distributions that can be used. Simple distributions, such as the exponential distribution function, do not verify the latter property. Jewitt (1988), however, has shown that the CDFC can be relaxed, provided that the Agent's utility function satisfies certain properties.

We show that the Gamma conditional distribution of loss verifies the first-order validation conditions of Jewitt (1988). We characterize the optimal insurance contract in presence of taxation using this distribution as well as a representation of the Agent's preferences also satisfying Jewitt's conditions. We then analyze how the tax affects equilibrium outcomes. More specifically, we analyze how increasing the Agent's share of the tax impacts his investment in self-protection/loss reduction, the Principal's average payouts, the Principal's profit and the overall social welfare. We simulate the above effects for different measures of risk aversion of the Agent and for different characteristics of his preferences and his likelihood to contract with alternative providers. We present the characteristics of the optimal contract and the impact of an increase in the insured tax share in case of a simple utility function. The closed form solutions to the problem emphasize the good properties of the Gamma distribution. Still, we check the robustness of our results by using alternative loss distributions: one proposed by Rogerson (1985) and one proposed by LiCalzi and Spaeter (2003). To the best of our knowledge, this is the most general, yet most detailed analysis of the impact on welfare of imposing a tax on insurance output.

Our results show that maximum effort of the insured is achieved when the insurer pays the full amount of the tax. An exception to the result occurs when losses happen to be uniformly distributed. In this case, the insured may increase his effort because he is unable to anticipate the size of his future losses and of his insurance tax amount. If the tax share paid by the insured were to increase, we find average insurance payments out to the insured would be higher, unless the provider recognizes the insured has made only marginal effort to self-protect and the insured is uncertain about keeping insurance with them. This suggests that the insurance provider would compensate other taxed insured through a higher coverage in order to keep their contract. Finally, the above increase in average payments would generate a decline in the provider's profit that is positively correlated to the insured tax share.

Under the assumption that losses are Gamma distributed, we find any increase in the tax share of an insured with low self-protection investment generates an overall decrease in welfare. Welfare is found to increase only for young and healthy individuals with constant marginal cost of effort and with a large number of other insurance contracts opportunities. When investment in self-protection is high, the effect on welfare is ambiguous. It is negative for insured whose marginal cost of effort is constant. It is positive however, when the cost of effort is high and convex. Furthermore, this positive effect is independent of the assumed distribution of losses. This result may be explained by a greater rise in the insured satisfaction when reducing his very costly effort. Finally, we find that more risk aversion, i.e. higher co-pay reinforces the insured's incentive to substitute care services for self protection. This last conclusion is reversed however, when uncertainty about future losses is higher.

The next section describes the model. In Section 3, we use a more general representation of the insured preferences to characterize the contract in presence of taxation and presents simulation results. Section 4 concludes.

2. The framework

The literature distinguishes three types of insurance models under the assumption of moral hazard. The first model analyzes the moral hazard on expenditures for self-protection. Self-protection is defined as an increase in the probability of a zero loss with no change in the conditional distribution of losses. In this model, a general insurance contract consists of a premium π and a payment function $I(x)$ where x is the loss. The probability of an accident depends on the care a undertaken by the individual on avoiding the loss. The insured loses x and receives $I(x)$. The decision variables are π, $I(x)$, and a. The second model analyzes the moral hazard on expenditures to reduce the size of a contingent loss (with no change in the probability of an accident). Conditional upon an accident, there are a finite number of loss sizes occuring with different probabilities. In this case, an insurance contract consists of a premium π and a finite number of possible payments contingent upon the loss size. The decision variables are π, $I_1, ..., I_n$ (where n is the total number of possible losses) and a. In this second model, the size of the loss depends on the care a undertaken by the individual on avoiding the loss.

In this paper, we use a more general framework similar to Holmström (1979). Unlike the models above, our framework allows for a distribution of losses with care affecting both the probability of a loss and the size of a loss (see also Winter, 2000). A risk averse Agent (the insured) whose demand for health care services is continuous, insures with a risk neutral Principal (the insurer or insurance provider). The insurance provider and the insured both have property rights to an uncertain income stream, i.e a possible loss from current wealth which depends on the care or effort on the part of the agent, to be taken in the future.

The insured, with initial wealth Y faces the risk of losing the amount of wealth x. The probability of a loss is represented by a density function $f\,(x|a)$; it is a function of the loss x for a given care a undertaken by the insured on avoiding the loss. The respective cumulative distribution function is given by $F\,(x|a)$ where $F\,(x|a)$ is absolutely continuous with respect to the same nonnegative measure for each a.This assumption can be illustrated by long term care insurance for example, where the probability of a loss is a direct function of the effort of the insured. In this case, a loss is declared "certified" if the insured becomes unable to perform at least two activities of daily living for an expected period of 90 days without substantial assistance from another person (i.e. eating, bathing...).

The insurer establishes a sharing rule, or contract, on how to share the random income stream. The contract describes the insurance payment to the agent $I(x)$ as a function of the loss incurred by the agent. The strategic variables here are $I(x)$ and a. The insurer collects an insurance premium from the insured in an amount π. This amount is assumed to be roughly equivalent to the average claim (Friedman, 1974), we set $\pi = \int_{x_0}^{\infty} xf(x|a,x_0)dx$ for $x \in [x_0, \infty)$. The insured, chooses an insurance plan giving him a transfer payment $I(x)$ in case of loss and a lifestyle with investment in self-protection a, of associated cost function $c(a)$, and has a separable von Neumann-Morgenstern event-independent utility function $U_A(I,\nu,a) = u(I(x),\nu) - c(a)$ where ν is a numeraire good, u is increasing concave and c is increasing convex (see Arnott, 1992 for further discussion and some examples). If the insured does not accept the proposed contract, he can still find an outside contract leaving him utility $\overline{U}$, so a constraint on the insurer's choice of I is that the agent's maximized expected utility must not be less than $\overline{U}$.

Each transfer payment to the insured $I(x)$ is subject to a tax rate t. The tax is shared by both parties: we denote by γ the share of the tax paid by the insured, and $(1-\gamma)$ the share paid by the insurer. Hence, $\gamma = 0$ when the insurer bears the entire tax burden, while $\gamma = 1$ when the insured does pay the full amount of the tax. Note that the marginal implications of this study would stay valid in the scenario where the loss value x was higher than a given number, say A, to be tax eligible. In this case, x would be replaced by $x' = x + A$.

The insurer's problem $[P]$ is to maximize his expected profit and can be written as:

$$\text{Maximize}_{I,a} \int_{x_0}^{\infty} [x - I(x) - (1-\gamma)tI(x)] f(x|a,x_0)dx \tag{2.1}$$

subject to the Participation Constraint

$$\int_{x_0}^{\infty} u\left[(1-t\gamma)\, I(x), \nu\right] f(x|a,x_0)dx - c(a) = \overline{U}, \tag{2.2}$$

and the Incentive Compatibility Constraint

$$\int_{x_0}^{\infty} u\left[(1-t\gamma)\, I(x), \nu\right] f_a(x|a,x_0)dx - c'(a) = 0. \tag{2.3}$$

We make three additional assumptions:

Assumption 1- For a given care level, the distribution of losses follows a Gamma distribution

$$x|a \sim \Gamma(p, a), \tag{2.4}$$

[2] where parameter p is a shape parameter controlling the scope of the distribution, while a is a scale parameter[3]. Density functions for different values of p and a are presented in Figures 1a-1b of Appendix 3 for a low effort level (a=1, dotted line), a medium effort level (a=5, thin continuous line) and a higher effort level (a=10, thick line). The Gamma distribution is particularly fitted to the case of insurance claims for its non-negative nature and the light nature of the tail of the distribution, as opposed to fire or liability insurance for example (see Klugman, Panjer and Willmot, 2004 for more details). Furthermore, this distribution verifies the "first-order approach" validation conditions Jewitt (1988, Theorem 1). Indeed,

$$\int_{x_0}^{y} F(x|a, x_0)dx = \frac{1}{\Gamma(p)} \int_{x_0}^{y} \gamma\left(p, \frac{x}{a}\right) dx,$$

where $\gamma(p, x/a) = \int_{x_0}^{\frac{x}{a}} t^{p-1}e^{-t}dt$ and $\frac{\partial\gamma\left(p, \frac{x}{a}\right)}{\partial a} = -\frac{x}{a^2}\left[\frac{x}{a}\right]^{p-1} e^{-\frac{x}{a}}$, is non-increasing convex in a for each value of y;

$$\int_{-\infty}^{+\infty} xF(x|a, x_0)dx = ap + x_0$$

is non decreasing concave in a, and

$$\frac{f_a(x|a, x_0)}{f(x|a, x_0)} = \frac{x - ap - x_0}{a^2} \tag{2.5}$$

is non-decreasing concave in x for each value of a. More details on the properties of this distribution are given in Table 1 of Appendix 1. We later compare our results to two alternative

[2]The density function for the gamma distribution is given by: $f(x|a, x_0) = f(x; p, a, x_0) = \frac{1}{a^p\Gamma(p)}(x - x_0)^{p-1} e^{-(x-x_0)/a}$, for $x \in [x_0, \infty)$, where $\Gamma(p) = \int_{x_0}^{\infty} e^{-t}t^{p-1}dt$.

[3]See Bose et al, 2011 for another application of this distribution to principal-agent models.

distributions (see Figures 1c-1d of Appendix 3): one proposed by Rogerson (1985) and one proposed by LiCalzi and Spaeter (2003). Some properties of these distributions are also reported in Table 1 of Appendix 1.

Assumption 2- We impose conditions on the shape of the insured utility function $u(I(x))$ using a general class of utility functions that satisfy Jewitt (1988) conditions validating the "first-order approach".

Proposition 1. *Let* $\omega(z) = u\left[u_I^{-1}\left(\frac{1}{z}\right)\right]$ *for all* $z > 0$, $x > 0$. *Assume that*

$$\omega(z) = K + zD,$$

where $D > 0$ *is independent of* z *and* K *is a positive constant. Then*

$$u(I(x)) = K + \sqrt{2DI(x)}. \tag{2.6}$$

Proof. See Appendix 2 □

The above utility function has two key properties: an upward slope, and a concave shape. (i) The upward slope implies that the person feels that more of $I\,(x)$ is better: a larger amount received yields greater utility, and the person always prefers a payment that is first-order stochastically dominant over an alternative one.[4] (ii) The concavity of the utility function implies that the person is risk averse: a sure amount is always preferred over a random amount having the same expected value. The risk attitude is directly related to the curvature of the utility function: risk neutral individuals have linear utility functions, risk seeking individuals have convex utility functions while risk averse individuals have concave utility functions. The degree of risk aversion can be measured by the curvature of the utility function. In the specification above, D controls for the degree of risk aversion: the higher D, the more concave $u(I)$ (note that $u_{II}(I) = -\frac{1}{4}\sqrt{\frac{2D}{I^3}}$). In other words, it takes more payments $I(x)$ to a low risk averse individual (D low) than to a high risk averse individual (D high) to provide the same amount of utility. In other words, the more risk averse the insured is, the more he agrees to spend in co-pay, hence possessing less insurance than a less risk

[4]In this case, the distribution function of one payment is preferred to another regardless of what $u()$ is, as long as it is weakly increasing.

averse individual. The constant K is linear in $u(.)$ but is independent of $I(x)$. It can be defined as $K := R + h\nu = R + h(Y - \pi)$, where R may be seen as a measure of the provider's reputation and h is the marginal utility of income. Hence, K gives a measure of percieved security or ease of mind of the insured when contracting with his insurance provider. This security feeling may come from his wealth or from the reputation of the company. For example, if the provider is a well known established company (R is high), the insured may not worry about getting the payment, which would increase his overall satisfaction.

Let

$$\alpha := 1 + (1-\gamma)t \text{ and } \beta := (1 - t\gamma)$$

for brevity. The Lagrangian corresponding to the Principal's Problem $[P]$ is

$$\begin{aligned}\mathcal{L} &= \int_{x_0}^{\infty} [x - \alpha I(x)]\, f(x|a,x_0)dx + \lambda \left[\int_{x_0}^{\infty} u\,[\beta I(x)]\, f(x|a,x_0)dx - c(a) - \overline{U}\right] \\ &\qquad + \mu \left[\int_{x_0}^{\infty} u\,[\beta I(x)]\, f_a(x|a,x_0)dx - c'(a)\right].\end{aligned}$$

Proposition 2. *Under assumptions (2.4) and (2.6),* $[P]$ *is characterized by*

$$\mu = \frac{a^2 \alpha c'(a)}{\beta D p}, \tag{2.7}$$

$$\lambda = \frac{\alpha}{D\beta}\left(\overline{U} + c(a) - K\right), \tag{2.8}$$

$$I\left(x\right) = \frac{1}{2D\beta}\left(\left(\overline{U} + c(a) - K\right) + \frac{c'(a)(x - ap - x_0)}{p}\right)^2. \tag{2.9}$$

Proof. See Appendix 2. □

Equation (2.9) shows that when reservation utility $\overline{U}$ is high and K is low, it is necessary for the company to increase the payout $I\left(x\right)$ in order to ensure acceptance of the contract by the insured. Moreover, the higher the share of the tax γ paid by the insured, the higher the insurance payout (Respectively, the higher the share paid by the insurance company, the lower the insurance payout). Note that insurance payouts are higher for individuals with higher self-protection costs and this increase in payment varies with the cost of self-protection. In other words, the contract is such that "sicker" individuals should receive higher payments.

Assumption 3: The cost of self protection/loss reduction is generally specified as:

$$c(a) = Aa^B \; A > 0, \; B \geq 1 \tag{2.10}$$

Lemma 3. *Under (2.4),(2.6) and (2.10), there exist at least one positive solution a.*

Proof. See Appendix 2. □

3. Comparative statics

This section is devoted to the analysis of the effect of a change in the insured tax share on variables such as his effort to invest in self-protection, the average payments of the insurance provider out to the insured, the provider's profit and the overall social welfare. We present these effects assuming three alternative specifications of the loss distribution.

3.1. Definitions.

Let us denote by $E(I)$, $E(\Pi)$ and $E(W)$, the expected payments, the expected profit and expected social welfare induced by $[P]$. Their expressions are as follows:

$$\begin{aligned}
E(I) &= \int \frac{D\beta}{2\alpha^2}\left(\lambda + \mu \frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2 f(x|a,x_0)dx \\
&= \frac{1}{2D\beta}\int \left(\left(\overline{U} + c(a) - K\right) + \frac{c'(a)}{\int \left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx} \frac{f_a(x|a,x_0)}{f(x|a,x_0)} \right)^2 f(x|a,x_0)dx \\
&= \frac{1}{2D\beta}\left[\left(\overline{U} + c(a) - K\right)^2 + \frac{\left(c'(a)\right)^2}{\int \left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx} \right]
\end{aligned}$$

where the third equality follows from the square expansion of the second equality and from the fact that $\int f_a(x|a,x_0)dx = 0$.

Let $E(x|a) \equiv \int x f(x|a,x_0)dx$. The expected profit of the insurer can be written:

$$E(\Pi) = \int [x - \alpha I(x)] f(x|a)\, dx = E(x|a) - \alpha E(I)$$

Finally, since welfare is

$$W(x) = x - \beta I(x) + (2D\beta I(x))^{1/2} - c(a),$$

$$\begin{aligned} E(W) &= E(x|a) - \beta E(I) + (2D\beta)^{1/2} \int I(x)^{1/2} f(x|a)\, dx - c(a), \\ &= E(x|a) - \beta E(I) + \int |M(x|a)|\, f(x|a)\, dx - c(a), \end{aligned}$$

where $M(x|a) = \overline{U} + c(a) - K + \frac{c'(a)}{\int \left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx} \frac{f_a(x|a,x_0)}{f(x|a,x_0)}$. Hence,

$$\begin{aligned} E(W) &= E(x|a) - \beta E(I) + \overline{U} - K \text{ if } |M(x|a)| > 0 \\ &= E(x|a) - \beta E(I) - \overline{U} - 2c(a) + K \text{ if } |M(x|a)| < 0 \end{aligned}$$

Consequently, we have:

$$\frac{\partial E(I)}{\partial \gamma} = \frac{1}{D\beta} \left[\overline{U} + c(a) - K + \frac{c''(a)}{\int \left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx} \right] c'(a) \frac{\partial a}{\partial \gamma} + \frac{t}{\beta} E(I),$$

$$\frac{\partial E(\Pi)}{\partial \gamma} = \frac{\partial E(x|a)}{\partial a} \frac{\partial a}{\partial \gamma} + tE(I) - \alpha \frac{\partial E(I)}{\partial \gamma},$$

and

$$\begin{aligned} \frac{\partial E(W)}{\partial \gamma} &= \frac{\partial E(x|a)}{\partial a} \frac{\partial a}{\partial \gamma} + tE(I) - \beta \frac{\partial E(I)}{\partial \gamma} \text{ if } |M(x|a)| > 0, \\ &= \frac{\partial E(x|a)}{\partial a} \frac{\partial a}{\partial \gamma} + tE(I) - \beta \frac{\partial E(I)}{\partial \gamma} - 2ABa^{B-1} \frac{\partial a}{\partial \gamma} \text{ if } |M(x|a)| < 0. \end{aligned}$$

As anticipated, the expected payments out to the insured increase with γ (decrease with $1-\gamma$) and decrease with D. They are higher when the insured has outside contracts possibilities, when his cost of investment in self-protection is higher and when his marginal cost of investment in self-protection is higher. Expected profits increase with average claims and decrease with average payments. Finally, expected welfare depends on how the cost of investment compares to the relative attractiveness of the insurance provider. More specifically, $c'(a)$ is the marginal cost of investment, $\frac{f_a(x|a,x_0)}{f(x|a,x_0)}$ measures the change in the probability of a loss when the insured invests in one more unit of self-protection, and $\int \left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx$ measures the variability of the change $\frac{f_a(x|a,x_0)}{f(x|a,x_0)}$. Consequently, welfare is lower following an increase in the insured tax share if the total cost of investing in self-protection (following a one unit change in investment) is lower than the relative benefit (independent of coverage) from contracting with the provider $K - \overline{U}$. It is higher otherwise.

Note that under the assumption of competitive insurance markets, solutions to $[P]$ remain unchanged[5]. However, expected welfare becomes:

$$\begin{aligned} E(W) &= tE(I) + \int |M(x|a)|\, f(x|a)\, dx - c(a), \\ &= tE(I) + \overline{U} - K \text{ if } |M(x|a)| > 0, \\ &= tE(I) - \overline{U} + K - 2c(a) \text{ if } |M(x|a)| < 0, \end{aligned}$$

and the effect of an increase in the insured's tax share on welfare would be

$$\begin{aligned} \frac{\partial E(W)}{\partial \gamma} &= t\frac{\partial E(I)}{\partial \gamma} \text{if } |M(x|a)| > 0, \\ &= t\frac{\partial E(I)}{\partial \gamma} - 2c'(a)\frac{\partial a}{\partial \gamma} \text{ if } |M(x|a)| < 0. \end{aligned}$$

3.2. **The Gamma Distribution.**

3.2.1. *General Results.*

The theorem below establishes a relationship between the amount of effort of the insured and his share γ of the tax.

Theorem 4. *Under assumptions (2.4),(2.6) and (2.10) there is a negative relationship between the share of the tax paid by the insured and his effort level:*

$$\frac{\partial a}{\partial \gamma} = \frac{-t^2p^2D}{\alpha^2 ABa^{B-2}\left(AB\,(2B-1)\,a^B\left(\frac{p^2}{2}+\frac{p}{B}+B+1\right)+p\left(\overline{U}-K\right)(B-1)\right)} \tag{3.1}$$

Proof. See Appendix 2. □

[5]The characterization of optimal competitive insurance contracts extends to any Pareto-optimal contracts, including the case where there is market power on the sellers side of the market (Winter, 2000).

Equation (3.1) shows that when K is sufficiently low or $\overline{U}$ is sufficiently large so that $\overline{U} \geq K$, increasing γ will decrease effort (respectively increasing $1-\gamma$ will increase effort). Note that the magnitude of the effect becomes smaller as it becomes more difficult to self-protect.

3.2.2. *An application with closed form solutions.* In a different context, Gupta and Viauroux (2009) consider an example of (2.6) where $K=0, D=2$ and $c(a)=a^2$, i.e.

$$U_A(I,a) = 2\beta^{1/2} I^{1/2} - a^2.^6 \tag{3.2}$$

This example is of particular interest for two reasons. First, it is representative of a low income population ($K=0$) with rather high coverage ($D=2$) and an increasing marginal cost of investment. Recall that one of the tax policy goals is to increase coverage for low income individuals. Second, solutions to $[P]$ under (3.2) have closed form solutions. A characterization of the optimal contract and of the policy implications are summarized in the proposition below[7]:

Proposition 5. *Under assumptions (2.4) and (3.2),*

(a) there exist an optimal amount of effort induced by the insurance premium strategy. Let $K := \frac{p\overline{U}}{3(p^2+p+6)}$ *and* $L := -\frac{p^2(1-t\gamma)}{2(p^2+p+6)(1+t-t\gamma)}$*, the optimal effort amounts to:*

$$a = \sqrt[3]{-L+\sqrt{L^2+K^3}} + \sqrt[3]{-L-\sqrt{L^2+K^3}},$$

(b) there is a negative relationship between the share of the tax assumed by the insured and his effort level:

$$\frac{\partial a}{\partial \gamma} = \frac{-t^2p^2}{\alpha^2\left[\frac{p^2(1-t\gamma)}{a} + 2a^2\left[p+p^2+6\right]\right]} < 0 \tag{3.3}$$

(c) the sign of $\frac{\partial E(I)}{\partial \gamma}$ *is ambiguous:*

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[6]Note that this utility function satisfies the properties of Von-Neumann and Morgenstern. Let $u = 2\beta^{1/2}I^{1/2} - a^2$and $v = u(J,a) = 2\beta^{1/2}J^{1/2} - a^2$. For $k(u) = Au + B$ for $A > 0$, B a real constant, we can easily show that $u > v$ implies (1) $k(u) > k(v)$ and (2) $k\left[\alpha u + (1-\alpha)v\right] = \alpha k\left[u\right] + (1-\alpha)k\left[v\right]$. Indeed, let $I > J$ so that $u > v$. From the monotony of function $x^{1/2}$, (1) is immediate. Moreover, $\alpha k\left[u\right] + (1-\alpha)k\left[v\right] = \alpha A 2\beta^{1/2}I^{1/2} + (1-\alpha)A2\beta^{1/2}J^{1/2} - Aa^2 + B = k\left[\alpha u + (1-\alpha)v\right]$. Hence, (2) is verified.

[7]The proof is available to the reader on request.

when $\overline{U} = 0$,

$$\begin{aligned} \frac{\partial E(I)}{\partial \gamma} &< 0 \textit{ if } \frac{2+t-\sqrt{1+4t}}{t} < \gamma < \frac{2+t+\sqrt{1+4t}}{t} \\ \frac{\partial E(I)}{\partial \gamma} &> 0 \textit{ otherwise,} \end{aligned} \tag{3.4}$$

when $\overline{U} > 0$, *a sufficient condition for* $\frac{\partial E(I)}{\partial \gamma}$ *to be positive is that*

$$\alpha^2 \left(1 - \frac{2\overline{U}a}{p\beta}\right) > 4t.$$

A sufficient condition for $\frac{\partial E(I)}{\partial \gamma}$ *to be negative is that*

$$\alpha \left[1 + p\overline{U} \frac{1 + \frac{\overline{U}}{a^2}}{\left[(p+4)a^2 + p\overline{U}\right]}\right] (\alpha + 2) < 4t.$$

(d) $\frac{\partial E(\Pi)}{\partial \gamma} < 0$; *i.e. the principal's expected profit is a decreasing function of* γ, *for all* $\gamma \in [0, 1]$.

(e) $\frac{\partial E(W)}{\partial \gamma} < 0$; *i.e. expected welfare is a decreasing function of* γ,*for all* $\gamma \in [0, 1]$.

Equation (3.3) shows that increasing the tax share paid by the insured (or decreasing the tax share paid by the insurer) decreases the insured effort to self-protect. And this impact on effort is all the more pronounced than the tax rate t is higher, than the insured' share of the tax is higher and than the optimal (current) effort level is lower. The intuition is the insured substitutes care services (medical for example) for self-protection when the insured pays more for insurance (through the tax). Equation (3.4) also shows that in absence of outside insurance competition ($\overline{U} = 0$), an increase in the insured's tax share increases the average coverage amount. This is true except for a small interval of γ, i.e. when the tax rate $t > \frac{3}{4}$ (since $\gamma \leq 1$). [8]

We find an increase in the insured's tax share decreases the company's profit. For sufficiently big output, welfare can be reduced by an amount increasing with the tax rate, with the insured' share of the tax paid by the insured and with the number of outside contract possibilities.

[8]The following table reports the cases of high taxes and high shares paid by the insured for which the impact of a change in γ would be negative; in all other cases, the impact is positive

t	0.75	0.8	0.85	0.9	0.95	0.99
$\gamma\vert_{\frac{\partial E(w)}{\partial \gamma}<0}$	$(0.84, 1)$	$(0.94, 1)$	$(0.88, 1)$	$(0.84, 1)$	$(0.8, 1)$	$(0.77, 1)$

3.3. **Alternative distributions.** In this section, we present additional conclusions under alternative conditional distributions of losses.

3.3.1. *Rogerson (1985) distribution:* Rogerson (1985) proposes the following cumulative distribution function:

$$F(x) = x^a, x \in [0,1] \tag{3.5}$$

Proposition 6. *Let* $Z = \left(\frac{1}{p} + B + \frac{1}{B} - 1\right)$. *Under assumptions (3.5),(2.6), and (2.10), we have:*

$$\frac{\partial a}{\partial \gamma} = \frac{-ABt^2 a^{B-1}\left(ABa^B Z + \overline{U} - K\right)}{Dp\beta^2\left(\frac{2}{(a+1)^3} + \frac{AB}{Dp}\frac{\alpha}{\beta}a^{B-2}\left[(B-1)\left(ABa^B Z + \overline{U} - K\right) + (AB^2a^B Z)\right]\right)} \tag{3.6}$$

Provided that $A \geq 0$ and $B \geq 1$, we see that increasing the insured's tax share(or lowering the share of the insurance provider) decreases investment in self-protection unless K becomes sufficiently large. This result is consistent with the result obtained with a Gamma distribution of losses.

3.3.2. *LiCalzi and Spaeter (2003) distribution:* One of the distributions satisfying the "first-order" approach proposed by LiCalzi and Spaeter (2003) has the cumulative distribution function:

$$F(x) = x + \frac{x - x^2}{1+a}, x \in [0,1] \tag{3.7}$$

Proposition 7. *Under assumptions (2.6), (2.10) and (3.7), we have the following result:*

$$\frac{\partial a}{\partial \gamma} = \frac{-Dpt^2 a^{1-2B}}{6\alpha^2 A^2 B^2 \begin{pmatrix} \frac{Dp\beta(2B-1)a^{-2B}}{6A^2B^2\alpha} + \frac{3a^2}{2p}\left(\frac{(a+1)}{a(a+2)} + \frac{2}{(a+1)} + \frac{3}{2}\ln\left(\frac{a}{a+2}\right)\right) \\ +\frac{a^3}{2p}\left(\frac{-a^2-2a-2}{a^2(a+2)^2} - \frac{2}{(a+1)^2} + \frac{3}{a(a+2)}\right) + 2B(a+1) + 2a \end{pmatrix}}. \tag{3.8}$$

Again here, we may observe a decrease in self-protection investment on the part of the insured when his tax share increases, provided that the denominator is positive.

4. SIMULATIONS

Tables 1 through 5 in Appendix 3 summarize the results of a change in the insured' share of the tax under optimal contract conditions, general preferences (2.6) and cost specifications (2.10). More specifically, they report simulation results of the effect of an increase in γ (or a decrease in $1-\gamma$) on the level of effort a invested in improving self-protection or loss reduction, on the expected insurance payment from the provider to the insured $E(I)$, on the expected profit of the insurance provider $E(\Pi)$ and on the overall expected welfare $E(W)$. We analyze these effects for a $[0,1]$ range of γ values and for various degrees of individuals risk aversion $D \in [1,10]$. We report the effects of the policy for two optimal levels of effort invested in self-protection: low ($a=1$) and high: ($a=10$). We assume three possible parametric specifications of the loss distribution as mentioned above. Note that the Rogerson (1985) distribution is a uniform distribution when $a=1$. Table 5 reports the results in case of competitive insurance markets. In all cases, we use a tax rate $t=40\%$, a unit cost of investment in self-protection/loss reduction $A=1$ and we analyze the effect of an increase in γ for different values of B and K.

In order to give an intuitive interpretation of the effect of the tax policy change, it is important to remember that:

(1) An increase in γ represents an increase in the insured' share of the tax and a decrease in the insurance provider' share of the tax $(1-\gamma)$.
(2) The more risk averse the insured (D high), the less insurance coverage, and the more co-pay spending.
(3) The cost of investing in self-protection or in loss reduction is controlled by parameters A and B. More specifically, B measures the convexity of the cost. When $B \geq 2$, the marginal cost of effort is increasing at an increasing rate. Hence, as investment in self-protection increases, it becomes increasingly difficult for the insured to protect himself against loss or damage. Typically, a young/uneducated and healthy individual may have a constant marginal cost of investment. The assumption that $B=1$ for example, where the cost is directly proportional to the effort could represent the cost of this individual. In this case, rather little is needed for the insured to reduce the occurrence of a damage. In contrast,

an elderly/unhealthy individual may find it increasingly difficult to protect himself against potential loss/accident as additional effort is made because the return to any investment becomes very small very quickly.

(4) Parameter $\overline{U}$ measures the reservation utility, i.e. the number of outside insurance contracts opportunities. A high value of $\overline{U}$ characterizes a "good" customer, while a low value of $\overline{U}$ characterizes a "dependent" customer whose personal situation prevent him from leaving easily their insurance provider.

(5) Finally, as is mentioned before, K gives a measure of percieved security or ease of mind of the insured at contracting with his insurance provider. The ease of mind may be explained by the insurer's reputation or by the fact that his wealth is sufficiently large to cover some unexpected losses. Hence, when K is large relative to $\overline{U}$, the insured has no incentive to leave the current company.

Simulation results are as follows. First, we find that any increase in the insured's tax share decreases his incentive to prevent loss; in other words, maximum effort on the part of the insured is achieved when the insurer pays the full amount of the tax. This conclusion is valid whether the insured is careless i.e. not much involved in protecting themselves against loss ($a = 1$, see Table 1 c.4-5, l.2), or not ($a = 10$, Table 1 l.8). Instead, the insured decides to rely more heavily on care services, substituting them for self-protection (Steward, 1994). Of course, each individual may well recognize that "excess" use of care makes the premium he must pay rise. No individual will be motivated to restrain his own use, however, since the incremental benefit to him for excess use is great, while the additional cost of his use is largely spread over other insurance holders, and so he bears only a tiny fraction of the cost of his use. It would be better for all insurance beneficiaries to restrain their use, but such a result is not forthcoming because the strategy of "restrain use" is dominated by that of "use excess care" : the insured find themselves in a "prisoners' dilemma" (see Pauly, 1974). Note also that this result generalizes the result from Section 3-2-2. An exception occurs when losses are uniformly distributed (Table 1, c.2) and the insured is unable to anticipate the size of future losses and their associated future tax amount. In this case, the fear of losing insurance coverage may trigger a sense of responsibility and increase the insured's effort. This is true especially when the insured is satisfied with his insurance provider and has no or little contract opportunities from alternative providers ($\overline{U} = 0, 10$; $K = 100$, Table 1, c. 2, l. 4-5).

We find any increase in γ triggers higher average insurance payments (Table 2) unless the provider identifies the insured has invested only marginally in self protection and is unsure about keeping insurance with them. Payments are lower for careless individuals ($a = 1$) with constant marginal cost of effort ($B = 1$) whose propensity to switch to a different provider ($\overline{U}$ close to or equal to K) is high. In this case, little can be done by the provider to keep the contract. This suggests the insurance provider may try to retain other individuals'contract by giving them rent (through higher insurance coverage). Note that an increase in average payments generates a decline in profits (Tables 3a-b).

The impact on overall welfare is ambiguous (Tables 4a-b).

Assuming losses are Gamma distributed, we find an increase in γ generates a decrease in welfare among careless insured ($a = 1$) and this negative impact on welfare remains in more specialized insurance markets (Table 4a, c.5), i.e. for a narrower loss distribution ($p = 100$). In this case, the overall decrease in provider's profit due to higher payments to the insured is greater than the gain in insured' satisfaction (from higher payments and lower costs of effort) and government revenue. Welfare is found to increase only for the population of young and healthy individuals with constant marginal cost of effort ($B = 1$) and many outside opportunities ($\overline{U} = 100, K = 0, 10$, Table 4a, c.4, l.6). The idea is that this population may not respond much to the tax policy, increasing insurance payments and insurance satisfaction (from low cost decreased effort) only marginally. Hence, welfare is mostly generated by government revenue. In case of more specialized market too, a positive welfare impact is found for individuals whose health/socio-economic situations make self-protection difficult ($B = 5$), triggering only a low response to the tax policy.

The effect of an increase in on welfare is ambiguous for more cautious insured ($a = 10$). It is negative for insured with constant marginal cost of effort ($B = 1$; Table 4b, c.4-5, l.3). It is positive, however, for individuals whose cost of effort is high and convex ($B = 2, 5$ for $p = 10$;Table 4b, c4., l2,l4; $B = 5$ for $p = 100$; Table 4b, c5., l4) and this positive effect is independent of the assumed distribution of losses. Intuitively, the marginal cost of effort is much higher than in case $a = 1$

above. Hence, reducing effort leads to a greater rise in satisfaction when $a = 10$ than it would if a were lower. Consequently, the significant increase in consumer satisfaction plus the added government revenue more than compensates the provider's loss in profit (due to higher payments). In a more specialized market ($p = 100, a = 10$), simulation results, under a Gamma distribution of losses, show increasing the insured's tax share decreases welfare unless it is very hard for the insured to self-protect ($B = 5$,Table 4b, c.5, l.4), in which case decreasing self-protection a slight amount substantially increases the insured's utility. In this case, the provider's losses are large relative to the increased government revenue and insured's utility.

Our results for effort level, average payments and profits are robust to (1) LiCalzi and Spaeter (2003) and to (2) Rogerson (1985) conditional distributions of losses when $a = 1$. However, in (1), an increase in the insured's tax share always increases welfare. In (2) and under a gamma distribution of losses however, an increase in welfare is observed only for healthy insured ($\overline{U} = 100, K = 0, 10$, Table 4b,c.1,l2) that feel unsure about keeping their insurance contract with the current provider. Intuitively, these insured respond poorly to the tax policy. Otherwise, we observe a decrease in welfare for insured with low cost of effort ($B = 1$, Table 4b, c.2,l.6), with no or little alternative insurance opportunity but who feel comfortable dealing with their current provider. In this case, a large increase in effort decreases satisfaction a lot.

Our simulations allow to go beyond the general results above and to conclude according to the type of insurance contract. For example, we find that as the insured becomes more risk averse and bears higher co-pay, any increase in the insured's tax share reduces the insured's effort to self protect further(the negative impact becomes more negative as D increases, see Table 1, c.3,4,5, l.2 and l.8). The intuition is the following: "If I pay for most of my losses, an additional payment from the tax reinforces my incentive to increase my use of care services as opposed to investing additional money for future protection". Here, less income is associated to more services. This is true under the Gamma and LiCalzi and Spaeter (2003) distributions of losses. The conclusion is reversed when the distribution of losses is uniform (Rogerson, 1985). In this case, we observe a positive income effect (the negative impact becomes less negative as D increases, see Table 1, c.2, l.2 and 8). In other words, "The higher my copay, the more likely a tax will give me the motivation to protect myself against loss and save on future taxes. Tax savings can be used toward future services." This may be explained by the presence of uncertainty about taxes on future insurance payments.

Next, we find an increase in the insured's tax share increases average payments less as insurance coverage decreases (the positive impact decreases with D, see Table 2, l.2 and l.8). This is intuitive since payments become smaller as risk aversion (and co-pay) increases. An exception exists under a Rogerson (1985) distribution of losses. Under this assumption, payments can be slightly higher for individuals with full coverage (D small) whose propensity to leaving the provider is high (Table 2, c.2, l.5). This result suggests, as above, that the provider gives a rent to "good customers". In fact, the insurance provider seems to face a trade off between retaining lots of contracts and retaining "good" contracts. For example, within the population of cautious insured ($a = 10$), the company only pays rent to healthy individuals ($B = 1$). The rent is given to a wider range of individuals ($B = 1, 2$) only when there is room for improvement in self-protection ($a = 1$). Finally, the company will not try to retain individuals whose cost of effort is very high ($B = 5$) as payments to these insured are likely to be higher.

The increase in payments above translates into a loss in provider's profit (Table 3a-b,l.2), of greater magnitude as gamma γ increases and lower magniture as the co-pay increases. Losses are higher for careless individuals ($a = 1$) with high copay (Table 3a, c.4-5, l.3,5,6,7) who are unsure about keeping their current provider. This is consistent with the above conclusion that the provider pays a rent to retain some contracts. This conclusion does not hold (Table 3b) for cautious individuals ($a = 10$) who find it difficult to protect themselves further $(B \geq 2)$; instead, we observe that profit is uniformly increasing with D: the more risk averse (the less insured), the more sensitive to the tax the insured is.

When markets are assumed competitive (Table 5), our results show an increase in the insured's share increases welfare and the positive effect decreases with the degree of risk aversion of the individual: less insurance means less tax revenue. An exception occurs for the subpopulation of insured, with linear cost of effort, who consider leaving their current provider. In this case, the positive impact of the tax decreases to a minimum (when D$\approx$ 6) then increases. Intuitively, the insured that are likely to leave (D high) their provider decrease their investment in self-protection/loss reduction to a greater extent; hence increasing their utility and the overall welfare gains.

5. Conclusion-Extensions

In this paper, we have evaluated the economic impacts of imposing a tax on insurance payment in presence of moral hazard. The analysis shows that the overall trend of current policies go toward greater welfare. It shows however that the policy may have an effect contrary to the intended one for some individuals, especially those who already invested highly in self-protection. This result opens to a broader discussion on unintended consequences of government policies in presence of moral hazard. A variety of programs help people who suffer the misfortune of poverty. Unemployment compensation pays people who suffer the misfortune of losing their jobs. Food stamps and public housing help the poor. Yet all these programs also suffer from problems of moral hazard. They increase unemployment and poverty. As the government expands a program to provide more aid to those in distress, it also encourages people to put themselves in distress. If people are paid to be poor, some will become poor. If people are paid to be unemployed, more will be unemployed. If it sometimes gives so little aid to those in distress that it provides little encouragement for people to put themselves in the situation, it then also provides little help for those in distress. Thus government programs that act to insure citizens against some misfortunes have a basic tradeoff that cannot be escaped. Greater efforts to help those in need also increase actions that are considered socially undesirable. Unintended consequences abound in the area of moral hazard.

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6. Appendix 1

Table 1: Some distribution properties

	Rogerson (85) $x \in (0,1)$	LiCalzi & Spaeter (03) $x \in (0,1)$	$Gamma$ $x \in (x_0, +\infty)$
F	$x^a, x \in (0,1)$	$x + \frac{x-x^2}{1+a}$	$\frac{\gamma(p,\frac{x}{a})}{\Gamma(p)}, p > 0$
f	ax^{a-1}	$1 + \frac{1-2x}{1+a}$	$\frac{1}{a^p\Gamma(p)}(x-x_0)^{p-1} e^{-\frac{(x-x_0)}{a}}$
$\int xf dx$	$\frac{a}{a+1}$	$\frac{3a+2}{6(a+1)}$	$ap + x_0$
$\int x^2 f dx$	$\frac{a}{a+2}$	$\frac{2a+1}{6(a+1)}$	$a^2p + (ap+x_0)^2$
$\frac{f_a}{f}$	$\frac{1}{a} + \ln x$	$-\frac{(1-2x)}{(1+a)(2+a-2x)}$	$\frac{x-ap-x_0}{a^2}$
$\frac{f_{aa}}{f}$	$\left(\frac{1}{a} + \frac{f_a}{f}\right)\ln x$	$\frac{-2}{(1+a)}\frac{f_a}{f}$	$\left(\frac{f_a}{f}\right)^2 - \frac{2}{a}\frac{f_a}{f} - \frac{p}{a^2}$
$\int \left(\frac{f_a}{f}\right)^2 f dx$	$\frac{1}{a^2}$	$\frac{-1+\frac{(a+1)}{2}\log\frac{a+2}{a}}{(a+1)^2}$	$\frac{p}{a^2}$
$\int \left(\frac{f_a}{f}\right)^3 f dx$	$\frac{-2}{a^3}$	$\frac{1}{a(a+1)(a+2)} + \frac{2}{(a+1)^3} + \frac{3\ln(\frac{a}{a+2})}{2(a+1)^2}$	$-\frac{p^3}{a^3}$
$\int xf_a dx$	$\frac{1}{(a+1)^2}$	$\frac{1}{6(a+1)^2}$	p
$\int x\left(\frac{f_a}{f}\right)^2 f dx$	$\frac{a^2+1}{a(a+1)^3}$	$\frac{\left[-2(3a^2+9a+7)+3(a+2)(a+1)^2\ln(\frac{a+2}{a})\right]}{12(a+1)^3}$	$-p^3a + \frac{(ap+x_0)p}{a^2}$

7. APPENDIX 2:

Proof of Proposition 1. If we define

$$u(y) := K + \sqrt{2Dy}, \quad y > 0 \tag{7.1}$$

with $D > 0$. We have

$$u_y(y) = \sqrt{\frac{D}{2y}} =: 1/z,$$

meaning that

$$\sqrt{\frac{2y}{D}} =: z \text{ or } \sqrt{2yD} =: Dz,$$

and hence

$$u(u_y^{-1}(1/z)) = u(y) = K + \sqrt{2Dy} = K + Dz.$$

□

Proof of Proposition 2. Let us assume that $y = \beta I$, then (7.1) becomes:

$$u(\beta I) := K + \sqrt{2D\beta I}, \quad I > 0 \tag{7.2}$$

$$u_I(\beta I) = \sqrt{\frac{\beta^2 D}{2I\beta}}.$$

Denote by $u_I^{-1}(x)$, the reciprocal function of $u_I(x) := \sqrt{\frac{\beta^2 D}{2x}}$. We have

$$u_I^{-1}(x) = \frac{\beta^2 D}{2x^2}. \tag{7.3}$$

Hence,

$$u\left[u_I^{-1}(\frac{1}{z})\right] = u\left(\frac{\beta^2 D}{2\left(\frac{1}{z}\right)^2}\right) = u\left(\frac{D\beta^2 z^2}{2}\right) = K + D\beta z. \tag{7.4}$$

Proposition 2 then follows from the maximization of $\mathcal{L}$ with respect to I, λ, μ.

Step 1. Pointwise maximization of $\mathcal{L}$ with respect to I yields

$$\frac{\partial \mathcal{L}}{\partial I} = -\alpha f(x|a, x_0) + \lambda u_I\left(\beta I(x)\right) f(x|a, x_0) + \mu u_I\left(\beta I(x)\right) f_a(x|a, x_0) = 0$$

or

$$\lambda + \mu \frac{f_a(x|a,x_0)}{f(x|a,x_0)} = \frac{\alpha}{u_I\left(\beta I(x)\right)}. \tag{7.5}$$

Since u_I is invertible, (7.5) gives:

$$I(x) = \frac{1}{\beta} u_I^{-1}\left(\frac{\alpha}{\lambda + \mu \frac{f_a(x|a,x_0)}{f(x|a,x_0)}}\right). \tag{7.6}$$

Finally, using (7.3) we have:

$$I\left(x\right) = \frac{D\beta}{2\alpha^2}\left(\lambda + \mu \frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2. \tag{7.7}$$

Step 2. Maximization of $\mathcal{L}$ with respect to λ, using (7.6) gives

$$\int_{x_0}^{\infty} u\left[u_I^{-1}\left(\frac{\alpha}{\left(\lambda + \mu \frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)}\right)\right] f(x|a,x_0)dx = \overline{U} + c(a).$$

Substituting (7.4), the preceding equality becomes

$$\int_{x_0}^{\infty}\left(K + \frac{D\beta}{\alpha}\left(\lambda + \mu \frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)\right) f(x|a,x_0)dx = \overline{U} + c(a),$$

and since

$$\int_{x_0}^{\infty} f_a(x|a,x_0)dx = 0, \tag{7.8}$$

solving for λ gives (2.8).

Step 3. Maximization of $\mathcal{L}$ with respect to μ yields

$$\int_{x_0}^{\infty} u\left[\beta I(x)\right] f_a(x|a,x_0)dx = c'(a).$$

Using (2.9) and (7.4), we have

$$\int_{x_0}^{\infty}\left[K + \frac{D\beta\lambda}{\alpha}\right] f_a(x|a,x_0)dx + \frac{D\beta\mu}{\alpha}\int_{x_0}^{\infty}\left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx = c'(a).$$

which, using (7.8) and Table 1, c.4, l.8 gives(2.7). Finally, the expression of (2.9) follows directly from substituting (2.7), (2.8) and (2.5) for μ, λ and $\frac{f_a}{f}$ in the expression of (7.7). $\square$

Proof of Lemma 3: Maximization of $\mathcal{L}$ with respect to a yields

$$\text{(7.9)} \qquad \int_{x_0}^{\infty}[x-\alpha I(x)]f_a(x|a,x_0)dx+\mu\left[\int_{x_0}^{\infty}u\left[\beta I(x)\right]f_{aa}(x|a,x_0)dx-c''(a)\right]=0,$$

Using (7.7),

$$\begin{aligned}
\text{(7.10)} & \int_{x_0}^{\infty}[x-\alpha I(x)]f_a(x|a,x_0)dx \\
= & \int_{x_0}^{\infty}xf_a(x|a,x_0)dx-\alpha\int_{x_0}^{\infty}\frac{D\beta}{2\alpha^2}\lambda^2\frac{f_a(x|a,x_0)}{f(x|a,x_0)}f(x|a,x_0)dx, \\
& -\alpha\int_{x_0}^{\infty}\frac{D\beta}{2\alpha^2}\mu^2\left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^3 f(x|a,x_0)dx-\alpha\int_{x_0}^{\infty}\frac{2D\beta}{2\alpha^2}\lambda\mu\left[\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right]^2 f(x|a,x_0)dx
\end{aligned}$$

where the first term simplifies from Table 1, l.10, c.4, the second term disappears from (7.8), the third term simplifies using Table 1, l.9, c.4, and the fourth term simplifies from Table 1, l.8, c.4. Finally, we obtain

$$\int_{x_0}^{\infty}[x-\alpha I(x)]f_a(x|a,x_0)dx=p+\frac{D\beta p^3}{2\alpha a^3}\mu^2-\frac{D\beta p}{\alpha a^2}\lambda\mu.$$

Moreover, we have

$$\begin{aligned}
& \int_{x_0}^{\infty}u(\beta I(x))\frac{f_{aa}(x|a,x_0)}{f(x|a,x_0)}f(x|a,x_0)dx \\
& =\int_{x_0}^{\infty}u(\beta I(x))\left[\left(\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2-\frac{2}{a}\frac{f_a(x|a,x_0)}{f(x|a,x_0)}-\frac{p}{a^2}\right]f(x|a,x_0)dx, \\
& =\int_{x_0}^{\infty}u(\beta I(x))\left(\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2-\frac{2}{a}c'(a)-\frac{p}{a^2}\left(\overline{U}+c(a)\right)
\end{aligned}$$

$$\begin{aligned}
= & \int_{x_0}^{\infty}\left(K+\frac{D\beta}{\alpha}\left(\lambda+\mu\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)\right)\left(\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2 f(x|a,x_0)dx-\frac{2}{a}c'(a)-\frac{p}{a^2}\left(\overline{U}+c(a)\right) \\
= & \int_{x_0}^{\infty}K\left(\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2 f(x|a,x_0)dx+\frac{D\beta p}{\alpha a^2}\lambda-\frac{D\beta p^3}{\alpha a^3}\mu-\frac{2}{a}c'(a)-\frac{p}{a^2}\left(\overline{U}+c(a)\right)
\end{aligned}$$

where the first equality is obtained substituting $\frac{f_{aa}(x|a,x_0)}{f(x|a,x_0)}$(see Table 1, l.7, c.4), the second equality uses (2.2) and (2.3), the third equality uses (7.2) and (7.7), the fourth equality is obtained by doing an expansion of the expression in the third equality and simplifying using Table 1, l.8 and l.9, c.4. Since

$$\int_{x_0}^{\infty} K\left(\frac{f_a(x|a,x_0)}{f(x|a,x_0)}\right)^2 f(x|a,x_0)dx = K\frac{p}{a^2}$$

(7.9) finally becomes

$$\frac{p}{\mu} - \frac{D\beta p^3}{2\alpha a^3}\mu + \frac{Kp}{a^2} - \frac{2}{a}c'(a) - \frac{p}{a^2}\left(\overline{U} + c(a)\right) = c''(a)$$

which, substituting (2.7) and (2.10) for μ and $c(a)$ respectively gives

$$\frac{p^2\beta D}{\alpha AB} - ABa^{2B-1}\left(\frac{p^2}{2} + \frac{p}{B} + B + 1\right) + p\left(K - \overline{U}\right)a^{B-1} = 0 \tag{7.11}$$

Note that equation (7.11) is of the form

$$d_1 + d_2a^{2B-1} + d_3a^{B-1} = 0$$

where d_1 d_2, d_3 are some constants, independent of a. For example, $d_1 = \frac{p^2\beta D}{\alpha AB}$, $d_2 = -AB\left(\frac{p^2}{2} + \frac{p}{B} + B + 1\right)$, and $d_3 = p\left(K - \overline{U}\right)$.

Denoting the left side of (7.11) by $f(a)$, $f : [0,\infty) \to \mathbb{R}$ is a continuous function. Consider (7.11). Since $f(0) = d_1 > 0$, the lemma follows from the intermediate value theorem if we prove that $f(a) < 0$ for large values of a. Let us rewrite $f(a)$ as

$$f(a) = d_1 + a^{2B-1}\left(d_2 + d_3a^{-B}\right).$$

Since $A > 0$, $B \geq 1$, for $a \to \infty$ we have

$$a^{2B-1} \to \infty \text{ and } d_2 + d_3a^{-B} \to d_2 < 0,$$

so that

$$a^{2B-1}\left(d_2 + d_3a^{-B}\right) \to -\infty,$$

and thus

$$f(a) = d_1 + a^{2B-1}\left(d_2 + d_3 a^{-B}\right) \to -\infty \qquad \square.$$

Proof of Theorem 4: Sign of $\partial a/\partial\gamma$. Recall that a is characterised by

$$\frac{p^2\beta D}{\alpha AB} - ABa^{2B-1}\left(\frac{p^2}{2} + \frac{p}{B} + B + 1\right) + p\left(K - \overline{U}\right)a^{B-1} = 0$$

Since that K and D are independent of γ, and noting that $\frac{\partial\left(\frac{\beta}{\alpha}\right)}{\partial\gamma} = \frac{-t^2}{\alpha^2}$, we have

$$-\frac{t^2p^2D}{\alpha^2AB} - AB\left(2B-1\right)a^{2B-2}\left(\frac{p^2}{2} + \frac{p}{B} + B + 1\right)\frac{\partial a}{\partial\gamma} + p\left(K - \overline{U}\right)\left(B-1\right)a^{B-2}\frac{\partial a}{\partial\gamma} = 0,$$

or

$$\frac{\partial a}{\partial\gamma} = \frac{-t^2p^2D}{\alpha^2ABa^{B-2}\left(AB\left(2B-1\right)a^B\left(\frac{p^2}{2} + \frac{p}{B} + B + 1\right) + p\left(\overline{U} - K\right)\left(B-1\right)\right)} < 0$$

Hence,

$$\frac{\partial a}{\partial\gamma} < 0 \text{ provided that } \overline{U} - K \geq 0 \qquad \square.$$

Proof of Proposition 6. Recall that maximization of $\mathcal{L}$ with respect to a yields

$$\int_{x_0}^{\infty}[x - \alpha I(x)]f_a(x|a,x_0)dx + \mu\left[\int_{x_0}^{\infty}u\left[\beta I(x),x\right]f_{aa}(x|a,x_0)dx - c''(a)\right] = 0,$$

Under a Rogerson (85) conditional distribution of losses, we have:

$$\int_{x_0}^{\infty}[x - \alpha I(x)]f_a(x|a,x_0)dx = \frac{1}{(a+1)^2} + \frac{D\beta\mu^2}{\alpha a^3} - \frac{D\beta}{\alpha a^2}\lambda\mu$$

where the first term of (7.10) simplifies from Table 1, l.10, c.2, the second term disappears from the fact that $\int_{x_0}^{\infty} f_a(x|a,x_0)dx = 0$, the third term simplifies using Table 1, l.9, c.2, and the fourth term simplifies from Table 1, l.8, c.2. Moreover, substituting $\frac{f_{aa}(x|a,x_0)}{f(x|a,x_0)}$using Table 1, l.7, c.2, (2.2) and (2.3) we have

$$
\begin{aligned}
&\int_{x_0}^{\infty} u(\beta I(x), x) \frac{f_{aa}(x|a, x_0)}{f(x|a, x_0)} f(x|a, x_0) dx \\
&= \int_{x_0}^{\infty} u(\beta I(x), x) \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} + \frac{1}{a} \right) \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} - \frac{1}{a} \right) f(x|a, x_0) dx, \\
&= \int_{x_0}^{\infty} u(\beta I(x), x) \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} \right)^2 f(x|a, x_0) dx - \frac{1}{a^2} \left(\overline{U} + c(a) \right)
\end{aligned}
$$

$$
\begin{aligned}
&= \int_{x_0}^{\infty} \left(C(x) + \sqrt{2D\beta I} \right) \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} \right)^2 f(x|a, x_0) dx - \frac{1}{a^2} \left(\overline{U} + c(a) \right) \\
&= \int_{x_0}^{\infty} \left(K + \frac{D\beta}{\alpha} \left(\lambda + \mu \frac{f_a(x|a, x_0)}{f(x|a, x_0)} \right) \right) \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} \right)^2 f(x|a, x_0) dx - \frac{1}{a^2} \left(\overline{U} + c(a) \right) \\
&= \left(K + \lambda \frac{D\beta}{\alpha} \right) \int_{x_0}^{\infty} \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} \right)^2 f(x|a, x_0) dx \\
&+ \mu \frac{D\beta}{\alpha} \int_{x_0}^{\infty} \left(\frac{f_a(x|a, x_0)}{f(x|a, x_0)} \right)^3 f(x|a, x_0) dx - \frac{1}{a^2} \left(\overline{U} + c(a) \right) \\
&= \left(K + \lambda \frac{D\beta}{\alpha} \right) \frac{1}{a^2} - 2\mu \frac{D\beta}{\alpha a^3} - \frac{1}{a^2} \left(\overline{U} + c(a) \right)
\end{aligned}
$$

where the first equality is obtained substituting $\frac{f_{aa}(x|a,x_0)}{f(x|a,x_0)}$(see Table 1, l.7, c.2), the second equality uses (2.2), the third and fourth equality use (2.6) under assumption (7.7), and the last equality simplifies using Table 1, l.8 and l.9, c.2.

Finally, the optimal amount of effort for self-protection/loss reduction is characterized by (7.9) which is

$$
\frac{1}{(a+1)^2} + \frac{D\beta\mu^2}{\alpha a^3} - \frac{D\beta}{\alpha a^2}\lambda\mu + \mu \left[\left(K + \lambda \frac{D\beta}{\alpha} \right) \frac{1}{a^2} - 2\mu \frac{D\beta}{\alpha a^3} - \frac{1}{a^2} \left(\overline{U} + c(a) \right) - c''(a) \right] = 0
$$

or, substituting for μ, becomes and simplifying gives:

$$
\frac{1}{(a+1)^2} - \frac{a\alpha c'(a)^2}{\beta D p^2} - \frac{\alpha c'(a)}{\beta D p} \left(\overline{U} - K + c(a) \right) - c''(a)\mu = 0
$$

Now, substitute for $c(a) = Aa^B$, $c'(a) = ABa^{B-1}$, $c''(a) = AB(B-1)a^{B-2}$, this equality simplifies to:

$$\frac{1}{(a+1)^2} - \frac{\alpha A^2 B^2 a^{2B-1}}{\beta Dp}\left(\frac{1}{p} + B - 1\right) - \frac{\alpha ABa^{B-1}}{\beta Dp}\left(\overline{U} - K + Aa^B\right) = 0$$

$$\frac{1}{(a+1)^2} - \frac{\alpha AB}{\beta Dp}a^{B-1}\left(ABa^B\left(\frac{1}{p} + B + \frac{1}{B} - 1\right) + \overline{U} - K\right) = 0$$

Since $\frac{\partial(\alpha/\beta)}{\partial\gamma} = \frac{-t\beta + t\alpha}{\beta^2} = \frac{t^2}{\beta^2}$, taking the derivative with respect to γ on both sides and solving for $\frac{\partial a}{\partial \gamma}$ gives (3.6) $\square$.

Proof of Proposition 7. We use a reasoning similar to the one above. Under a LiCalzi and Spaeter (2003) conditional distribution of losses, we have:

$$\begin{aligned}
&\int_{x_0}^{\infty}[x - \alpha I(x)]f_a(x|a, x_0)dx \\
= \quad & \frac{1}{6(a+1)^2} - \frac{D\beta}{2\alpha}\mu^2\left(\frac{1}{a(a+1)(a+2)} + \frac{2}{(a+1)^3} + \frac{3\ln(\frac{a}{a+2})}{2(a+1)^2}\right) - \frac{D\beta}{\alpha}\lambda\mu\left(\frac{-1 + \frac{(a+1)}{2}\ln\frac{a+2}{a}}{(a+1)^2}\right) \\
= \quad & \frac{1}{(a+1)^2}\left[\frac{1}{6} - \frac{D\beta}{2\alpha}\mu^2\left(\frac{(a+1)}{a(a+2)} + \frac{2}{(a+1)} + \frac{3}{2}\ln(\frac{a}{a+2})\right)\right]
\end{aligned}$$

where the first equality follows from (7.10) and from the distribution properties (see Table 1, c.3, l.8 and l.9). Furthermore,

$$\begin{aligned}
&\int_{x_0}^{\infty} u(\beta I(x), x)\frac{f_{aa}(x|a, x_0)}{f(x|a, x_0)}f(x|a, x_0)dx \\
= \quad & \frac{-2}{(1+a)}\int_{x_0}^{\infty} u(\beta I(x), x)\left(\frac{f_a}{f}\right)f(x|a, x_0)dx = \frac{-2}{(1+a)}c'(a)
\end{aligned}$$

where the first equality uses Table 1, c.3, l.7 and the second equality follows from (2.3). Dividing by μ, substituting for the cost function derivatives, dividing by a^{B-2} and simplifying gives:

$$\frac{1}{(a+1)^2}\left[\frac{1}{6} - \frac{D\beta}{2\alpha}\mu^2\left(\frac{(a+1)}{a(a+2)} + \frac{2}{(a+1)} + \frac{3}{2}\ln(\frac{a}{a+2})\right)\right] + \mu\left[\frac{-2}{(1+a)}c'(a) - c''(a)\right] = 0$$

$$\frac{Dp\beta a^{1-2B}}{6A^2B^2\alpha} - \frac{a^3}{2p}\left(\frac{(a+1)}{a(a+2)} + \frac{2}{(a+1)} + \frac{3}{2}\ln(\frac{a}{a+2})\right) - B(a+1)^2 - a^2 + 1 = 0.$$

Now let us take the derivative on both sides, we obtain (3.8) $\square$.

8. APPENDIX 3:

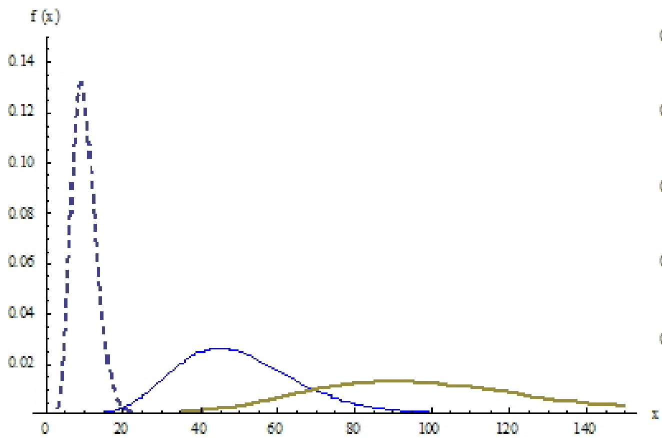

Fig.1a: Gamma p.d.f. for $p = 10$, $a = 1, 5, 10$

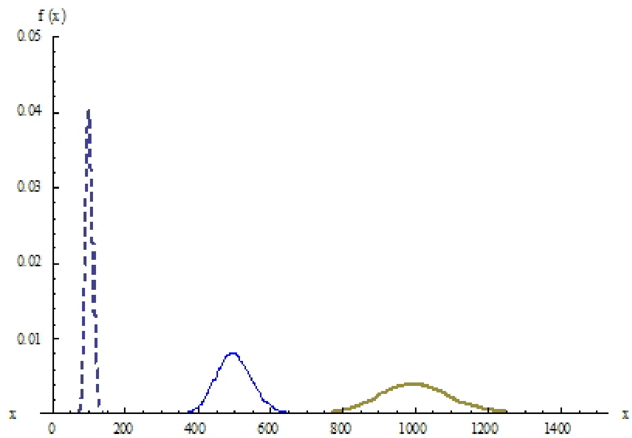

Fig.1b: Gamma p.d.f. for $p = 10$, $a = 1, 5, 10$

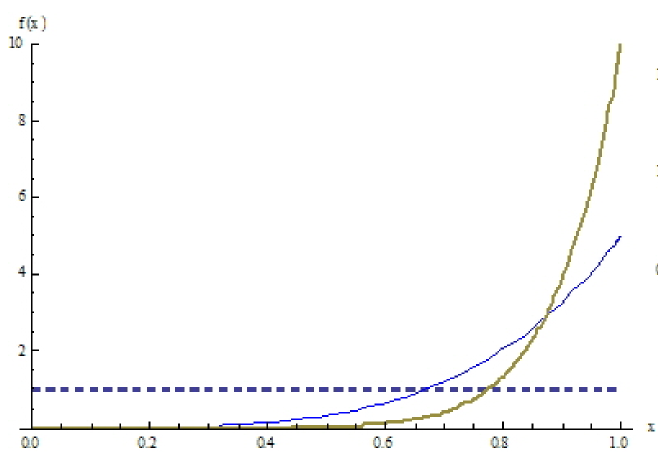

Fig.1c: Rogerson p.d.f. for $p = 10$, $a = 1, 5, 10$

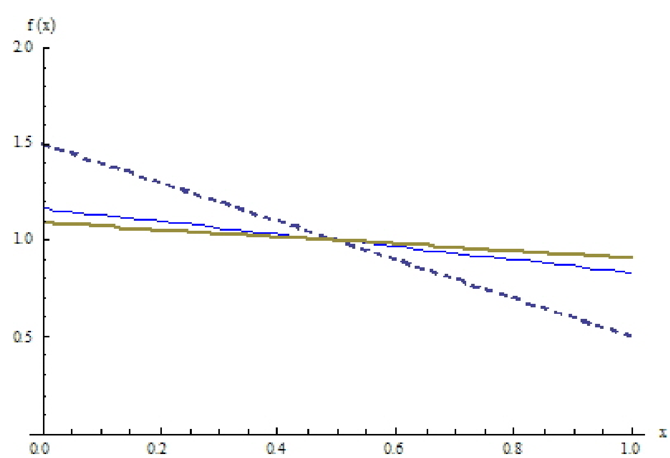

Fig.1d: LiCalzi and Spaeter p.d.f. for $p = 10$, $a = 1, 5, 10$

Table 1: Effect of tax on effort level

$\frac{\partial a}{\partial \gamma}$, for $a = 1$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100$* $K = 0, 10, 100$ $B = 1, 2, 5$	< 0 $\downarrow$ in γ; $sl \uparrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D
Exceptions				
$\overline{U} = 0; K = 10$ $B = 1$	$< 0\|_{D<4.5}$; $> 0\|_{D>5.5}$ $\uparrow$ in γ; $\downarrow$ in $D\|_{D<4.5}$ $\uparrow$ in $D\|_{D>5.5}$	NA	NA	NA
$\overline{U} = 0; K = 10; B = 2$ $\overline{U} = 0; K = 100; B = 5$ $\overline{U} = 10; K = 100; B = 5$	> 0 $\uparrow$ in γ; $\downarrow$ in D	NA	NA	NA
$\overline{U} = 0, 10; K = 100$ $B = 1, 2$ for Rogerson $B = 2, 5$ for Gamma	< 0 $\downarrow$ in γ; $\downarrow$ in D	NA	> 0 $\uparrow$ in γ; $\uparrow$ in D	NA
$\frac{\partial a}{\partial \gamma}$, for $a = 10$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100$* $K = 0, 10, 100$ $B = 1$	< 0 $\downarrow$ in γ; $\uparrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U} = 0, 10, 100$* $K = 0, 10, 100$ $B = 2, 5$	< 0 $\downarrow$ in γ; $\longrightarrow$ in D	NA	NA	NA
Exceptions				
$\overline{U} = 0, 10; K = 100$ $B = 1$	< 0 $\downarrow$ in γ; $\longrightarrow$ in D	NA	NA	NA

Table 2: Effect of tax on average payouts

$\frac{\partial E(I)}{\partial \gamma}$, for $a = 1$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 1, 2, 5$	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D
Exceptions				
$\overline{U} = 0, K = 0$ $\overline{U} = 10, K = 10$ $\overline{U} = 100, K = 100$ $B = 1$	NA	$> 0\|_{D<3}$, $< 0\|_{D>5}$ $\downarrow_{D=3}\uparrow$ in γ $\downarrow$ in D	$> 0\|_{D<1}$ $< 0\|_{D>1}$ $\uparrow$ in γ $\downarrow$ in D	$> 0\|_{D<1}$ $< 0\|_{D>1}$ $\uparrow$ in γ $\downarrow$ in D
$\overline{U} = 0, K = 10$ $B = 1$	$> 0\|_{D<4.5}$, $< 0\|_{D>5.5}$ $\uparrow$in γ; $\downarrow$ in D	NA	NA	NA
$\overline{U} = 0, K = 10$ $B = 2$	$> 0\|_{D>.5}$, $< 0\|_{D<.5}$ $\uparrow$ in $\gamma\|_{D>.5}$; $\uparrow^1\downarrow$ in D	NA	NA	NA
$\frac{\partial E(I)}{\partial \gamma}$, for $a = 10$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 1, 2, 5, 10$	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D

Table 3a: Effect of tax on provider's profit ($a=1$)

$\frac{\partial E(\Pi)}{\partial \gamma}$, for $a=1$	$Rogerson$ $p=10$	$LiC.\&Sp.$ $p=10$	$Gamma$ $p=10$	$Gamma$ $p=100$
$\overline{U}=0,10,100$* $K=0,10,100$ $B=1,2,5$	<0 $\downarrow$ in γ; $\uparrow$ in D	<0 $\downarrow$ in γ; $\uparrow$ in D	<0 $\downarrow$ in γ; $\uparrow$ in D	<0 $\downarrow$ in γ; $\uparrow$ in D
Exceptions				
$\overline{U}=0, K=0$ $\overline{U}=10, K=10$ $\overline{U}=100, K=100$ $B=2$, for Rogerson $B=1$, for LiC&Sp $B=1,2,5$ for Gamma	$<0\|_{D>2}$ $>0\|_{D<2}$ $\uparrow$ in γ; $\downarrow$ in D	>0 $\uparrow$ in $\gamma\|_{D\neq 0}$; $\uparrow^3\downarrow$ in D	<0 $\downarrow$ in γ; $\downarrow$ in D	<0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U}=0,10, K=100$ $\overline{U}=100, K=10$; $B=1$	NA	NA	NA	<0 $\downarrow$ in γ; $\uparrow^7\downarrow$ in D
$\overline{U}=0, K=10$ $B=2$	$<0\|_{D>2.5}$ $>0\|_{D<2.5}$ $\downarrow$ in γ; $\downarrow_3\uparrow$ in D	NA	<0 $\downarrow$ in γ; $\uparrow^7\downarrow$ in D	<0 $\downarrow$ in γ; $\uparrow^1\downarrow$ in D
$\overline{U}=10, K=0$ $B=1,2$	NA	NA	<0 $\downarrow$ in γ $\uparrow^3\downarrow$ in $D\|_{B=1}$	<0 $\downarrow$ in γ $\downarrow$ in $D\|_{B=1}$, $\uparrow^3\downarrow$ in $D\|_{B=2}$
$\overline{U}=0, K=10$ $B=1$	$<0\|_{D<5.5}$ $\downarrow$ in γ; $\uparrow\downarrow$ in D $<0\|_{D>7}$ $\uparrow$ in γ; $\longrightarrow$ in D	$<0\|_{D<4}$, $>0\|_{D>5}$ $\downarrow$ in γ; $\uparrow$ in D	<0 $\downarrow$ in γ; $\uparrow^2\downarrow$ in D	<0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U}=0, K=0$ $B=5$	$<0\|_{\gamma>0.3}$, $>0\|_{\gamma<0.2}$ $\downarrow$ in γ; $\uparrow$ in D	NA	NA	NA

Table 3b: Effect of tax on provider's profit ($a = 10$)

$\frac{\partial E(\Pi)}{\partial \gamma}$,for $a = 10$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 1, 2, 5$	< 0 $\downarrow$ in γ; $\uparrow$ in D	< 0 $\downarrow$ in γ; $\uparrow$ in D	< 0 $\downarrow$ in γ; $\uparrow$ in D	< 0 $\downarrow$ in γ; $\uparrow$ in D
Exceptions				
$\overline{U} = 0, K = 0$ $\overline{U} = 10, K = 10$ $B = 1$	> 0 $\downarrow$ in γ; $\downarrow$ in D	NA	< 0 $\downarrow$ in γ; $\uparrow^{2.5}\downarrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U} = 0; K = 10$ $B = 1$	> 0 $\downarrow$ in γ; $\downarrow$ in D	NA	NA	< 0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U} = 10; K = 0$ $B = 1$	$< 0\|_{\gamma > 0.4} > 0\|_{\gamma < 0.3}$ $\downarrow$ in γ; $\uparrow$ in D	NA	< 0 $\downarrow$ in γ; $\uparrow^{5}\downarrow$ in D	< 0 $\downarrow$ in γ; $\uparrow^{1}\downarrow$ in D
$\overline{U} = 100; K = 100$ $B = 1$	> 0 $\downarrow$ in γ; $\downarrow$ in D	NA	< 0 $\downarrow$ in γ; $\uparrow^{2.5}\downarrow$in D	< 0 $\downarrow$ in γ; $\uparrow^{6}\downarrow$in D
$\overline{U} = 0, 10, 100;$ $K = 0, 10$ $\overline{U} = 100, K = 100$ $B = 2$	$< 0\|_{\gamma > 0.7} > 0\|_{\gamma < 0.6}$ $\downarrow$ in γ; $\downarrow$ in D	NA	NA	NA
$\overline{U} = 0, 10, K = 100$ $B = 2$	$< 0\|_{\gamma > 0.05} > 0\|_{\gamma < 0.05}$ $\downarrow$ in γ; $\uparrow$ in D	NA	NA	NA
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 5$	$< 0\|_{\gamma > 0.3} > 0\|_{\gamma < 0.2}$ $\downarrow$ in γ; $\downarrow$ in D	NA	NA	NA

Table 4a: Effect of tax on welfare ($a=1$)

$\frac{\partial E(W)}{\partial \gamma}, a=1$	$Rogerson$ $p=10$	$LiC.\&Sp.$ $p=10$	$Gamma$ $p=10$	$Gamma$ $p=100$
$\overline{U}=0,10,100$ $K=0,10,100$ $B=1,2,5$	>0 $\uparrow$ in γ; $\downarrow$ in D	>0 $\uparrow$ in γ; $\downarrow$ in D	<0 $\downarrow$in γ; $\downarrow$ in D	<0 $\downarrow$in γ; $\downarrow$ in D
Exceptions				
$\overline{U}=0; K=10$ $B=1$	$<0\|_{D<4.5}, >0_{D>5.5}$ $\uparrow$ in γ, $\uparrow\downarrow$ in $D\|_{D<4.5,W_1}$ $\uparrow$ in γ, $\downarrow$ in $D\|_{D>5.5,W_1}$ $\uparrow$ in γ, $\uparrow$ in $D\|_{D<4.5,W_2}$ $\uparrow$ in γ, $\downarrow$ in $D\|_{D>5.5,W_2}$	NA	NA	NA
$\overline{U}=0; K=10$ $B=2$	>0 $\uparrow$ in γ, $\downarrow$ in $D\|_{W_1}$ $\downarrow_3\uparrow$ in γ, $\downarrow_4\uparrow$ in $D\|_{W_2}$	NA	NA	NA
$\overline{U}=0,10; K=100$ $B=1,2$	<0 $\downarrow$in γ; $\uparrow$ in D	NA	NA	NA
$\overline{U}=100; K=0,10$ $B=1$	NA	NA	>0 $\uparrow$in γ; $\downarrow$ in D	NA
$\overline{U}=100; K=0,10$ $B=5$	NA	NA	NA	$>0\|_{D<5}, <0\|_{D>5}$ $\downarrow$in γ; $\downarrow$ in D

Table 4b: Effect of tax on welfare ($a = 10$)

$\frac{\partial E(W)}{\partial \gamma}, a = 10$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 2$	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ $sl \downarrow$ in $D\|_{W_1}$ $sl \uparrow$ in $D\|_{W_2}$	< 0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 1$	NA	NA	< 0 $\downarrow$ in γ; $\downarrow$ in D	< 0 $\downarrow$ in γ; $\downarrow$ in D
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 5$	NA	NA	> 0 $\uparrow$ in γ; $\longrightarrow$ in D	> 0 $\uparrow$ in γ; $\longrightarrow$ in D
Exceptions				
$\overline{U} = 0; K = 10, 100$ $B = 2$	NA	NA	< 0 $\downarrow$ in γ; $\downarrow$ in D	NA
$\overline{U} = 0, 10; K = 100$ $B = 1$	< 0 $\downarrow$ in γ; $\uparrow$ in D	NA	NA	NA
$\overline{U} = 10; K = 100$ $B = 2$	NA	NA	NA	$< 0\|_{D<6}$; $> 0\|_{D>6}$ $\downarrow$ in γ; $\downarrow$ in D
$\overline{U} = 10; K = 0$ $B = 1$	NA	NA	$< 0\|_{D>\frac{5}{2}}$; $> 0\|_{D<\frac{5}{2}}$ $\downarrow \gamma\|_{D>\frac{5}{2}}$; $\downarrow \gamma\|_{D<\frac{5}{2}}$ $\downarrow$ in D	NA
$\overline{U} = 100; K = 0, 10$ $B = 1$	NA	NA	> 0 $\uparrow$ in γ; $\downarrow$ in D	NA

Table 5: Effect of tax on welfare in case of competitive markets ($a = 10$)

$\frac{\partial E(W)}{\partial \gamma}, a = 10$	$Rogerson$ $p = 10$	$LiC.\&Sp.$ $p = 10$	$Gamma$ $p = 10$	$Gamma$ $p = 100$
$\overline{U} = 0, 10, 100^*$ $K = 0, 10, 100$ $B = 1, 2, 5$	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D	> 0 $\uparrow$ in γ; $\downarrow$ in D
$\overline{U} = 0$ $\overline{U} = 10$ $\overline{U} = 100$ $K = 0$ $K = 10$ $K = 100$ $B = 1$ $B = 1$ $B = 1$	NA	NA	> 0 $\uparrow$ in γ; $\downarrow_6\uparrow$ in D	> 0 $\uparrow$ in γ $\downarrow_6\uparrow$ in D
$\overline{U} = 0; K = 10$ $B = 1$	NA	NA	> 0 $\uparrow$ in γ; $\downarrow_1\uparrow$ in D	> 0 $\uparrow$ in γ; $\uparrow$ in D