U.S. recession forecasting using probit models with asset index predictor variables

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November, 2016

Abstract

A considerable amount of research exists using probit models to predict a binary recession indicator variable. The single predictor variable which has demonstrated the greatest predictive potential to date is the *yield spread*, the difference between the yields on long-term and short-term Treasury securities. The most recent two recessions (2001 and 2008) were preceded by historic asset bubbles, the first due to inflated stock prices and the second due primarily to inflated home prices. Two metrics of asset bubbles in combination were found to be remarkably good indicators of these latest recessions with an out-of-sample predictive power exceeding that of the yield spread. The Cyclically Adjusted Price Earnings Ratio (CAPE) was found to give both a better in-sample fit and better predictive power than then S&P 500.

"All models are wrong. Some are useful."

Attributed to George Box

"The only function of economic forecasting is to make astrology look respectable."

Attributed to John Kenneth Galbraith

"I don't even know what a bubble means. These words have become popular. I don't think they have any meaning."

Eugene Fama (2010)

"In the garden, growth has it seasons. First comes spring and summer, but then we have fall and winter. And then we get spring and summer again."

Chance, the gardener, *Being There* (film, 1979)

Introduction

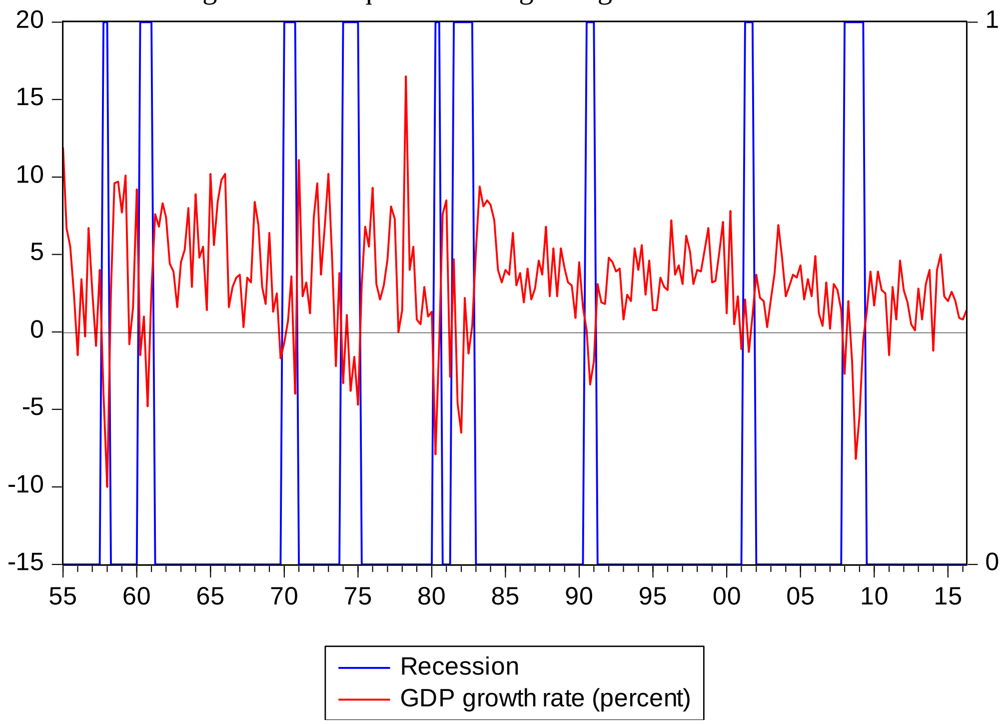
Predicting Business cycles presents the greatest forecasting problem in macroeconomics in term of both its importance and its challenge. Recessionary periods can bring considerable hardship to many, especially the unemployed and their dependents. Advanced notice of impending recession is of interest to households, firms and investors. Given that business cycle stabilization policy is generally seen as a responsibility of government, accurate forecasts of business cycle peaks and troughs are of particular interest to central bankers and policy makers. Incorrectly timed counter cyclical policy actions may be ineffective or even aggravate fluctuations.

The National Bureau of Economic Research (NBER) is commonly accepted as the official arbiter of recession dates in the United States and started publishing business cycle dates in 1929. Determining business cycle turning points is now the responsibility of the Business Cycle Dating Committee of the NBER, started in 1978. The NBER developed a methodology for the empirical analysis of business cycles using the definition suggested by Burns and Mitchell (1946): "business cycles are co-movements of several macroeconomic variables which determine the turning points, peaks and troughs, in aggregate economic activity." Recessions start just following a peak and end at a trough, representing the period of broadly declining economic activity, and expansions follow a trough and end at a peak. These cycles are reoccurring but not periodic. The NBER states that "[a] recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in production, employment, real income, and other indicators." It is "marked by widespread contraction in many sectors of the economy …, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales."

It is popular in the financial press to define a recession as beginning with two consecutive quarters of decline in GDP. While the NBER considers GDP the single best measure of aggregate economic activity, it considers the GDP definition to be too narrow a measure of economic activity to reliably date recessions. So for example, the 2001 recession did not include two consecutive quarters of negative growth. Nevertheless, declines in GDP are closely correlated to recession periods (see graph). Since GDP estimates by the Bureau of Economic Analysis (BEA) of the Department of Commerce are only available quarterly, the BCDC uses monthly estimates of GDP by the private forecasting firm of Macroeconomic Advisers.

Figure 1

GDP growth and recessions for the period of this study, 1955Q1 to 2016Q2. Note that not all recessions begin with two quarters of negative growth.



BCDC uses a variety of monthly indicators with particular emphasis on personal income less transfer payment (in real terms) and employment. Also there are two indicators with coverage primarily of manufacturing and goods: industrial production and the volume of sales and the manufacturing and wholesale-retails sectors (adjusted for price changes). The NBER employs “no fixed rule about which other measures contribute information to the process.” (NBER)

The BCDC approach is retrospective. They wait until sufficient data are available to avoid major revisions. In determining the onsets of recessions they wait until they are confident that “even in the event that activity begins to rise again immediately, it has declined enough to meet the criterion of depth. As a result, we tend to wait to identify a peak until many months after it actually occurs.”(NBER)

Typically, the BCDC declares the start of recessions after the fact anywhere from 6 to 18 months, “long enough so that the existence of a recession is not at all in doubt.” (NBER) Likewise, for the troughs: 6 to 18 months. Peak and trough months are determined and the BCDC takes no stand on the date within the month. (This lag in reporting is relevant in the real-time application of *dynamic probit models*, not to be confused with *autoregressive probit models*, both discussed later in this paper, where a lagged value of the recession indicator is used as a predictor variable). NBER announced the December 2007 peak a year later. Over the past 30 years, NBER announcement have been made from 6 to 20 months after a corresponding peak or trough.

Table 1 NBER recession dates for the sample period

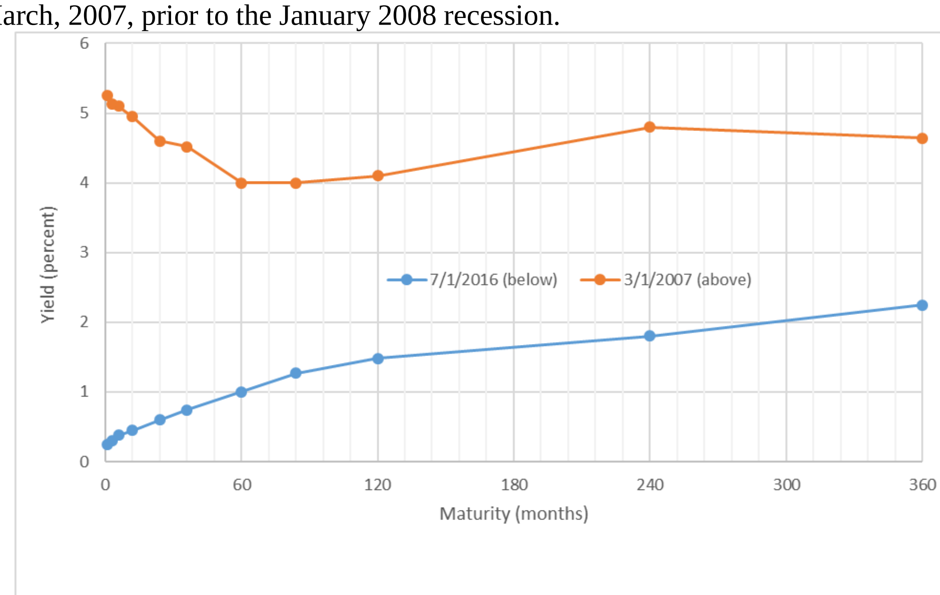
Peak	Trough
August 1957	April 1958
April 1960	February 1961
December 1969	November 1970
November 1973	March 1975
January 1980	July 1980
July 1981	November 1982
July 1990	March 1991
March 2001	November 2001
December 2007	June 2009

Earlier attempts at forecasting economic cycles examined continuous or interval dependent variables such as GDP, consumption and employment (Stock and Watson, 2003). Much recent research has focused on predicting recessions as discrete states rather that making quantitative estimations of future economic activity. Predicting recessions with a binary recession indicator dates back at least to the 1990's. Stock and Watson (1991) focused on recessions as discrete events and thus predicted whether the economy was in a recession state or not rather than forecasting continuous or interval variables such as GDP growth. Estrella and Hardouvelis (1991) used a static probit model (discussed later in this paper) and many studies in recent decades have used a binary indicator models for forecasting recessions (Dueker, 1997; Estraella and Mishkin, 1997; Chauvet and Potter, 2005; Wright, 2006; Kauppi and Saikkonen, 2008; Nyberg, 2010; Ng, 2011; Fossati, 2015). An advantage of a binary recession indicator variable is that it isolates the start and duration of recessions. On the other hand, studies of continuous or interval dependent variables such as economic growth, mix information on the magnitudes with that of the timing of the onset and duration of the economic cycles in their measures-of-fit. Estrella and Mishkin (1998) point out that "[t]he discrete dependent variable also sidesteps the problem of spurious accuracy associated with quantitative point estimates of, for example, future real gross domestic product (GDP) growth" and that models forecasting continuous economic variables suffer more from endogeneity problems than those giving probabilities. Hamilton (1989) presents an empirical analysis that suggests that the economy evolves differently within distinct discrete states, some corresponding to recessionary and expansion periods.

Many recession leading indicators have been considered by researchers over recent decades. One in particular has been repeatedly validated as perhaps the best predictor variable, the difference between the yield on short and long-term U.S. Treasury securities, called the *yield spread*, *yield curve* or *term spread* (Estrella and Trubin 2006). “Yield curve” refers to the plot of yields versus maturity time span, which typically slopes upward depicting lower yield on short-term securities and increasing yields as the maturities increase. A flattening or inversion of the slope signals the onset of a recession anywhere from two to six quarters ahead. I will not discuss the mechanisms connecting the yield curve to recessions other than to say that they are spooky and mysterious.

Figure 2

A recent and typical yield curve from July, 2016 and a partially inverted yield curve from March, 2007, prior to the January 2008 recession.



The literature on information in the yield spread related to forecasting recessions goes at least as far back as Kessel, 1956. Fama (1984) discusses the relationship between the term structure and business cycles. Stock and Watson (1991) use a version of the yield spread as well as a number of other leading indicators including housing permits, manufacturing orders, indices of exchange rates, employment and financial variables other that the yield spread. They suggest that important omitted variables such stock prices and consumer sentiment might improve the models. Estrella and Hardouvelis (1991) discuss the power of the yield curve as a predictor of economic activity, and specifically that the yield curve flattened and then inverted in 1989 in advance of the 1990 recession. In addition to the term spread, Estrella and Mishkin (1997) consider interest rates spreads, stock prices, monetary aggregates, macro indicators, and leading indices of the Commerce Department and use a novel measure of predictive power, Estrella's pseudo R^2, which is used in subsequent literature (Kauppi, 2008; Nyberg, 2010; Fossati, 2012; Ng, 2012). As with much of the literature, they note that the 1990 recession seemed uniquely difficult to forecast. They find that overfitting is a serious problem in macroeconomic prediction and that the in-sample and out-of-sample performance can differ greatly. The yield curve again shines as a forecasting variable. Dueker (1997) uses a dynamic probit model in which a lag of the depended indicator is used as a predictor variable in the model, a "probit analogue of adding a lagged dependent variable to a linear regression model" and used a probit model with Markov switching as well. His predictors included the Commerce Department's index of leading indicators, M2 money growth, and change in Standard and Poor's 500 index of stock prices. Moneta (2003) shows the effectiveness of yield spreads in the Euro Zone and that the yield curve outperforms the OECD Composite Leading Indicator for the Euro Area, the quarterly growth rate of the stock price index, and GDP growth especially beyond one quarter. Chauvet and Potter (2005) use a model that takes into account the predictive instability of the yield curve, using breakpoints across business cycles.

Wright (2006) finds the federal funds rate in addition to the term spread improves the predictive power of the probit models (but ruins out-of-sample result in my experience). Wright uses the RMSE to measure out-of-sample fit. Kauppi and Saikkonen (2008) used dynamic models and iterated forecasting and again relied on the interest rate spread "as the driving predictor." Katayama (2010) uses a combination of 33 macroeconomic indicators with a 6-month horizon including a combination of the term spread, change in the S&P 500 index and the growth rate of non-farm employment. Katayama compares several cumulative probability distribution functions in place of the cumulative normal function of the probit model, preferring the Laplace CDF. Nyberg (2011) includes domestic and foreign yields spread and stock market returns in a dynamic probit model. He also uses a dynamic *autoregressive* model using a lagged value of the latent variable and the recession indicator dependent variable as predictors as well as an interactive term of the lagged dependent variable and the spread variable. His out-of-sample period includes the 2001 and 2008 recessions. He examines German recessions as well as U.S. recession and confirm the domestic term spread as an important predictor.

Eric Ng (2011) incorporates various recession risk factors, including financial market expectations, credit or liquidity risks, asset price variables (as in this paper) and macroeconomic fundamentals in addition to the term spread in advanced dynamic probit models. He finds that while dynamic models outperformed static models, and dynamic autoregression models were only marginally better still. Static models did as good a job at forecasting the onset of recessions.

Serena Ng and Wright (2013) focus on the Great Recession which they say is unlike most postwar recession in "being driven by deleveraging and financial market factors" and how it differs from those driven by supply and monetary policy shocks. To this they attribute the failure of economic models and predictors, such as the term spread, that work well otherwise, to predict this most recent recession. Fossati (2015) also found that "models that use only financial indicators exhibit a large deterioration in fit after 2005." He obtains substantial improvements over Estrella and Mishkin (1998), Wright (2006) and Katayama (2010). He finds as well that models using macro factors are more robust to revisions in indicator values ex post compared to vintage data.

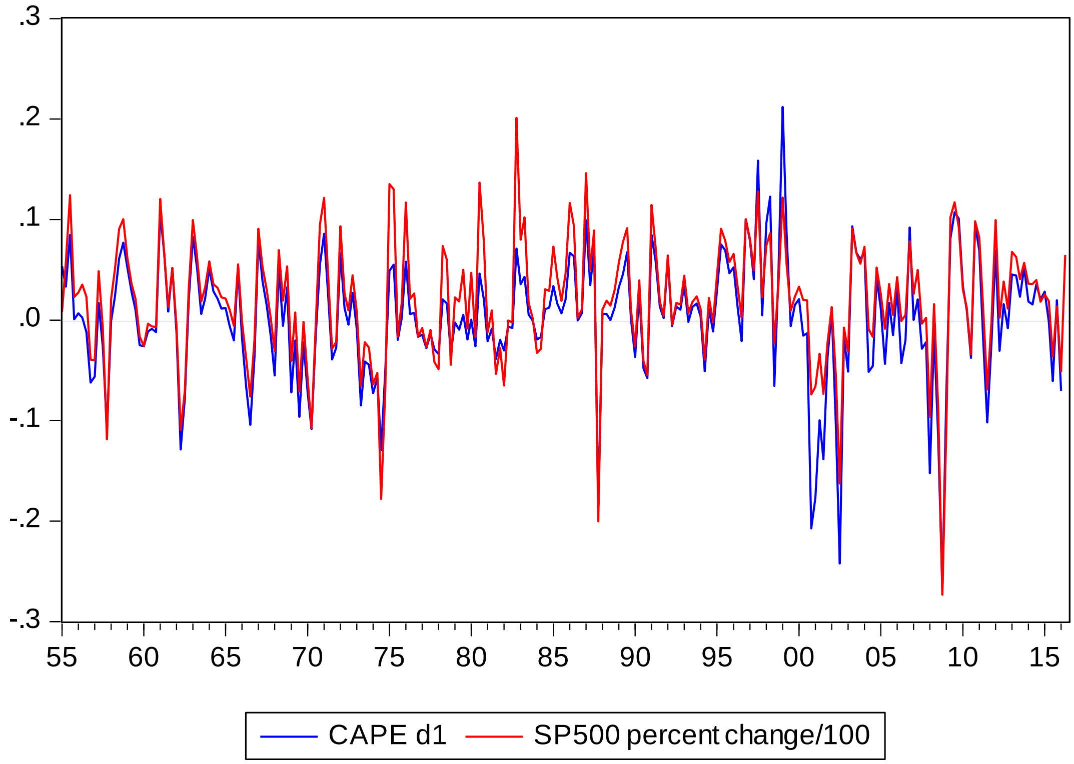
Data

Both of the two most recent recessions were preceded by unsustainable asset bubbles much greater than any others in the sample period 1955 to 2016 (see Figure 5 and Figure 7). Some previous literature (Ng, 2011; Fossati,2012; Christiansen et al, 2013) considered asset effects as precedents of recessions in their models. Part of the contribution of this paper is to test whether pre-2000 recession data can be used to estimate a model forecasting this century's recessions. The special character of the 2001 and 2008 recessions puts into question whether information from prior history can be used to forecast these two recent recessions. This objective motivates my choice of variables to include an equity index and a home price index, and of my choice of the in-sample period to include approximately the last four and a half decades of the 20^{th} century, and an out-of-sample to start from 2000q1. Ng (2011) uses an inflation adjusted S&P equity index and a Case-Shiller Home Price Index. But to the best of my knowledge, these two effects were not considered in isolation or jointly in comparison to the yield spread or in combination with the yield spread alone as I do in this study. Furthermore, the equity price indices used in previous literature (Ng, 2011; Fossati,2012; Christiansen et al, 2013) are not a measures of the general price level as compared to historical earnings and so do not necessarily reflect market overvaluation as does the Cyclically Adjusted Price Earnings ratio (also known as the Shiller PE Ratio), which I use here. I chose only the yield spread and two asset measures as predictors so as to determine the relative effectiveness of the information in the term spreads as compared to that in the asset indices and to specifically focus on these variables alone in the general interests of parsimony. Our principle interest is the out-of-sample performance. In-sample fit can always be improved by adding more variables but this does not necessarily lead to better out-of-sample performance, and in fact, frequently makes it worse. A contribution of this paper is to compare the predictive power of the asset variables to the benchmark yield spread.

As mentioned, for equity assets, the specific variable I chose is the Cyclically Adjusted Price Earnings ratio (CAPE). It is the ratio of the S&P index level to the ten-year inflation adjusted earnings of the index. Since the intrinsic value is the present value of the long-run returns, the CAPE can thus be seen as an indicator of an overpriced market or “bubble”. The CAPE reflects deviations of equity prices from their fundamental values. A change in the CAPE would reflect market a deviation away from, or return toward and theoretical steady state of “correct” valuation and it is the changes in the CAPE that in theory would be reflected in changes in the state of the overall macroeconomy. Hence, I use the first difference of the CAPE in the model. A plausible mechanism of action of the CAPE on the recession state is that the wealth effect of inflated equity prices creates high aggregate demand which abruptly sinks when the market quickly corrects. The reduced aggregate demand is accompanied by general economic doldrums which constitute the recession. The CAPE may as well be a reflection of economic conditions antecedent to recession. The actual CAPE variable used in the calculations in this study is normalized by dividing by the average over the sample period.

Instead of the CAPE, other authors (Ng, 2011; Fossati,2012; Christiansen et al, 2013) use market indices such as the S&P 500 or PE ratio. However, the CAPE’s use of a longer run earnings average smooths the volatility of short-term earnings and medium term general economic cycles and hence better reflects the market’s long-term earnings in comparison to current prices. The CAPE is a better metric of *overvaluation* of stocks since it is a ratio of contemporary price to long-term historical returns. A comparison of the two variables as to their effectiveness in the model is made below in the section on model selection. Figure 3 *CAPEd1 and change in S&P 500.*, below, compares the percent change in the S&P 500 to the first difference of the CAPE variable, hereafter designated CAPEd1. The two variables generally track each other closely, as would be expected, but at times one may deviate more severely than the other. Note in particular, that the swings of the CAPE in 2000, prior to the 2001 recession, are more extreme than those of the changes in the S&P 500.

Figure 3 CAPEd1 and change in S&P 500.



The variable I chose as metric of housing bubble is the Case-Shiller Home Price Index (CS). It is available with a two-month lag and is frequently subject to revision. The sizes of the revisions are of little consequence to this study and my results are robust to these ex-post alterations in the CS. Many other economic indicators are determined and announced with considerable delay and are often revised after the fact as well. For example, GDP figures are available about three months after the end of a quarter, and as mentioned, announcements of the beginning of recessions are, at best, available 6 months after the fact. Both the CAPE and the variable reflecting the term spread are available in real time, are precise, and generally are not subject to ex-post revisions as is typical of macroeconomic indicators. The variable CS used in this study is normalize by dividing by the average value over the sample period.

Robert Shiller (2013) found that “[h]ome prices look remarkably stable when corrected for inflation. Over the 100 years ending in 1990 — before the recent housing boom — real home prices rose only 0.2 percent a year, on average” yet house prices rose by about 90% between 1996 and 2007 (Shiller 2007). By contrast real rent increased by only about 5% in the same period. Hence the “dramatic price increase [of house prices] is hard to explain, since economic fundamentals do not match up with the price increases.” (Shiller, 2007) On the basis of this, one might say that the CS reflects home price deviations from some theoretical intrinsic values since it compares general market prices to their historic prices. It compares current prices to a sort of long-term average using repeat sale prices and so reflects whether the market is in a normal or abnormal price state. Like the equity market, the market for houses is not efficient but subject to animal spirits sometimes resulting in irrational exuberance. As in the case of the CAPE, a high CS produces a wealth effect and a sudden drop in the CS can foretell an economic slowdown.

My dynamic model uses the quarter *t*-2 recession state, which is not known in real-time. However, for forecasting the start of a recession on an ongoing basis, one might assume that the current and last quarters are non-recession ones. It is generally recognized that the economy is in a recession a quarter or two into one. Nonetheless, the dynamic model here assumes unrealistically, that the current recession state and previous ones are known. If the dynamic model does not show a distinct advantage over the static models, even given this unrealistically perfect information up to date, then under realistic circumstances it could only fare worse.

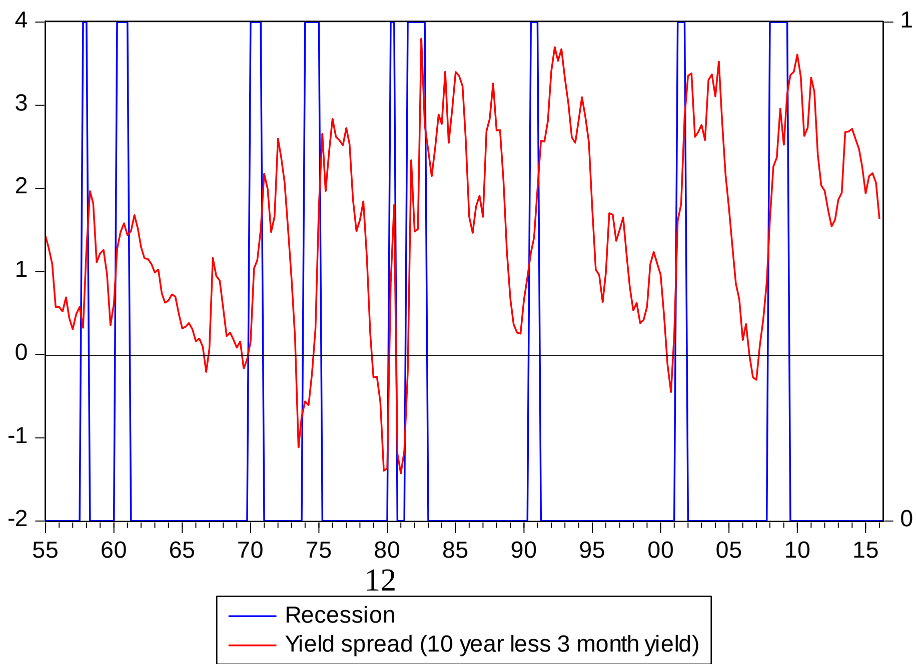
I use quarterly data of recession states and follow Kauppi and Saikkonen (2006) in defining a quarter as the first quarter of a recession if its first month or the proceeding quarter’s second or third moth is classified as the NBER business cycle peak and classify a given quarter as the last quarter of a recession period if its second or third month or the subsequent quarter's first month is classified as the NBER business cycle trough. For reasons mentioned, the in-sample period is from 1955q1 to 1999q4.

For a term spread variable, market analysts often use the difference between the ten-year and two-year Treasury rates. Some academic researchers have used the spread between the ten-year Treasury and federal funds rate. Most common in the literature using probit models to predict recessions is the spread between the 10-year bond yield and the 3-month Treasury bill rate which I adopted here, in part to be able to make comparison between our findings here and those in the literature. I will call this variable *spread*. Advantages and disadvantages of different rates are discussed in Estrella and Trubin (2006). The data for the Treasury securities are readily available at the Federal Reserve Economic Data website or the U.S. Treasury Department website.

A glance at the graphs of each of the three variables I use shows information each contains with respect to recessions. In the graph for spread, note small or negative spread prior to all the recessions in the sample.

Figure 4

The yield spread



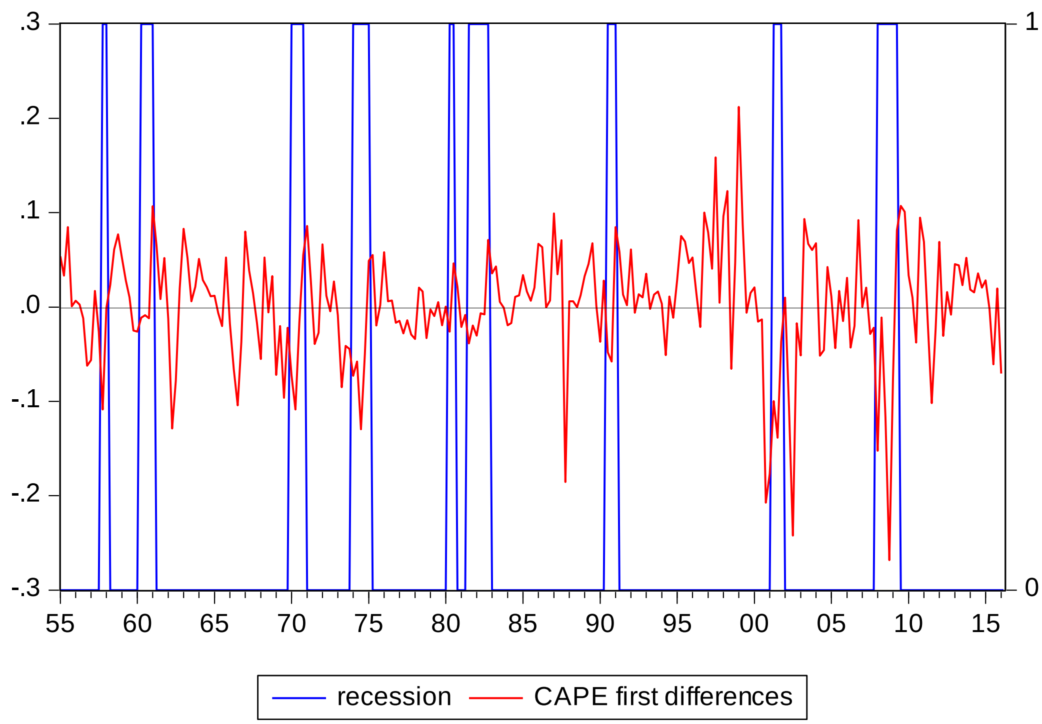
A graph of CAPE and recessions over the data set is given below. Note distinct peak prior to 2001 recession as well as smaller peaks before the 1970, 1973, and 2008 recessions.

Figure 5

Cyclically Adjusted Price Earnings Ratio (CAPE)

Figure 6

First-differences of the Cyclically Adjusted Price Earnings Ratio



A graph of CS and recessions over the data set is given below. Note distinct peak prior to 2008 recession as well as peaks before he 1973, 1980, and 1990 recessions.

Figure 7

Case-Shiller Home Price Index

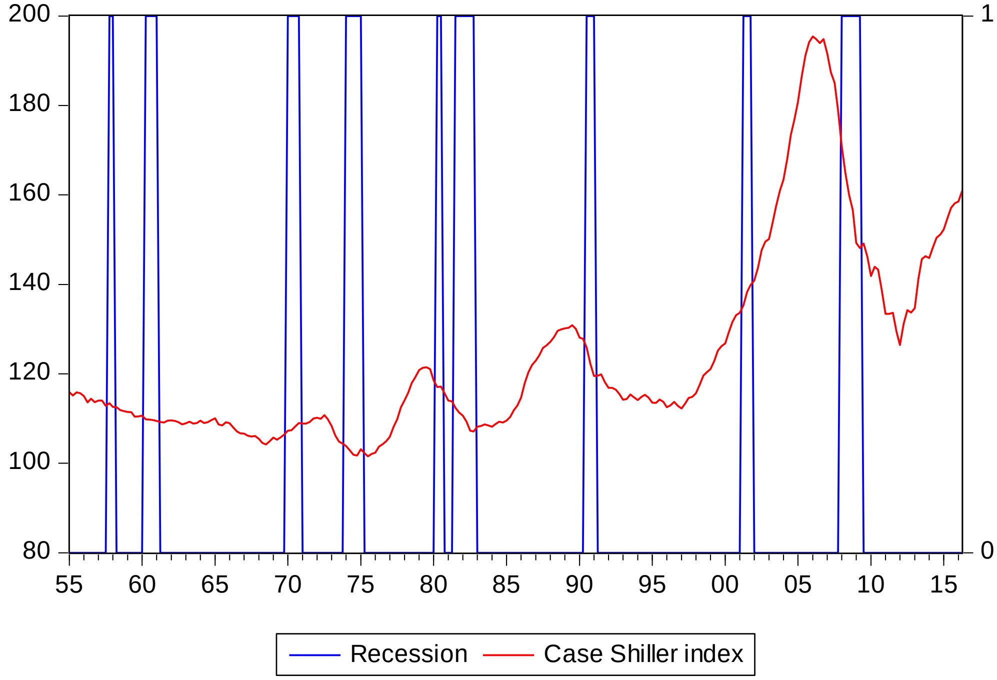
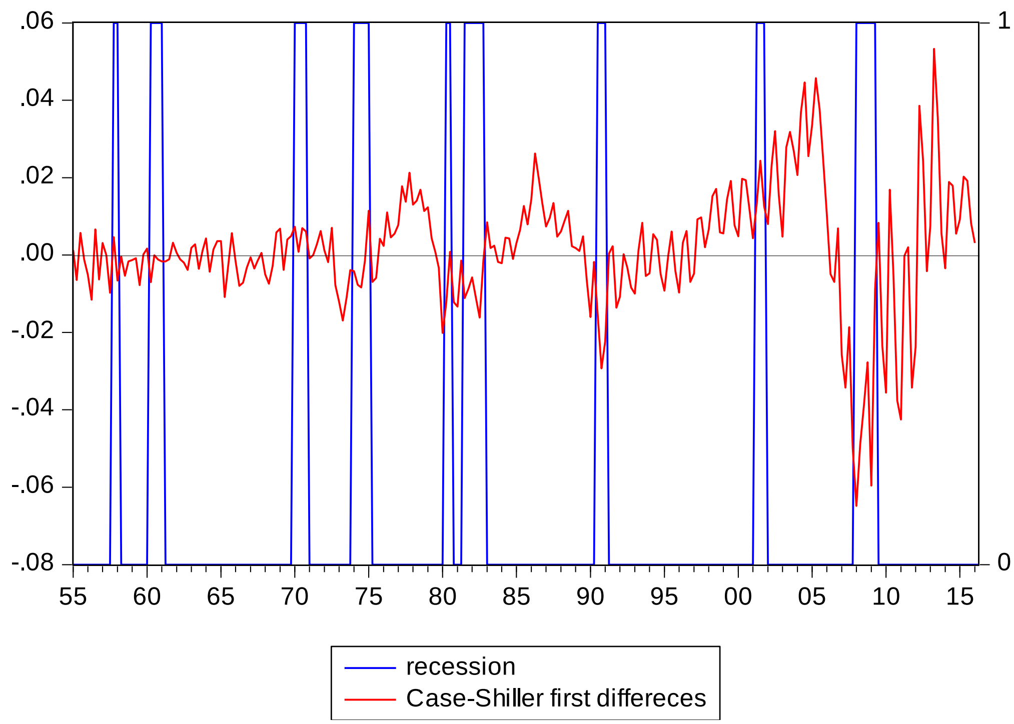


Figure 8

First-differences of the Case-Shiller Home Price Index



Econometric model

In consideration of models, let y_t be the binary indicator variable which equals one at time *t* if and only if the economy in a recession state at time *t*. We wish to forecast the time series $\{y_t\}_{t=1}^{T}$

where

$$y_t = \begin{cases} 1, & \text{the economy is in a NBER determined recession at time } t \\ 0, & \text{the economy is not in a recession at time } t \end{cases} .$$

We will let *r* be the recession recognition lag, the number of time periods that pass before the recession state of the economy is known, and *s* be the data availability lag for the explanatory variable *x*.

Conditional on the information set $\Omega_{t-h} = \{y_{t-j}, y_{t-j-1}, ..., x_{t\text{-}k}, x_{t\text{-}k\text{-}1}, ...\}$ where $j \geq h + r \;\; k \geq h + s$, available at time $t - h$, y_t has a Bernoulli distribution

$$y_t | \Omega_{t-h} \sim B(p_t)$$

where $p_t = P_{t-h}(y_t = 1)$, the conditional probability. Because y_t takes values 0 or 1, its expected value is $E_{t-h}(y_t) = p_t$. If $x_{t\text{-}k}$ is the vector of explanatory variables in the static probit model, a linear function of the exogenous explanatory variables gives the value of an unobserved variable

$$\pi_t = \alpha + x'_{t-k}\beta$$

from which the probability is determined from the cumulative normal distribution function

$$P_{t-h}(y_t = 1) = \Phi(\pi_t) = \int_{-\infty}^{\pi_t} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du .$$

Katayama (2010) considered other cumulative distribution functions in place of Φ and found that some right-skewed distributions increased the proportion correctly predicted. Lowing the threshold of a “correct” prediction from 50% can have a similar effect.

The parameters of the model are estimated by the criteria of a maximum likelihood function

$$L(y,\pi)_t = \prod_{t=1}^{T} \Phi(\pi_t)^{y_t}[1-\Phi(\pi_t)]^{1-y_t} \ ,$$

or equivalently a maximum log likelihood function

$$\mathcal{L}(y,\pi)_t = \sum_{t=1}^{T} \{y_t \log \Phi(\pi_t) + (1-y_t)\log[1-\Phi(\pi_t)]\} \ .$$

Because the static model neglects the serial dependence of the recession variable it may produce misleading or implausible recession probability forecasts. If the previous state of the economy is not included in the model, it does not take into account the autocorrelative structure of the binary time series. The solution proposed by Dueker (1997) is to remove the serial correlation by using a lag of the recession indicator, thus using information in the autocorrelative structure of the dependent variable, a probit analogue of adding a lagged dependent variable to a linear regression model. This so-called *dynamic probit model* uses past values of the recession variable as a predictor variable on the right-hand-side of the equation for the latent variable function.

$$\pi_t = \alpha + x'_{t-k}\beta + \delta y_{t-l}$$

for a single lag of *y* and

$$\pi_t = \alpha + x'_{t-k}\beta + \sum_{l=h+r}^{q} \delta_l y_{t-l}$$

for multiple lags. In application, it is only feasible to include y_{t-j} for $j \geq h+r$, where *r* is the number of periods of the delay in recognition of a recession state.

Kauppi and Saikkonen (2008) consider the *autoregressive probit model* where the latent variable is given as

$$\pi_t = \alpha + \beta' x_{t-k} + \gamma \pi_{t-1}$$

having a lag of itself as a predictor. Combined with the dynamic model this gives the dynamic autoregressive specification

$$\pi_t = \alpha + \beta' x_{t-k} + \delta y_{t-l} + \gamma \pi_{t-1} \ .$$

Ng (2011) finds that the dynamic autoregressive specification adds little to the fit and predictive power of the dynamic model alone. We will not consider autoregressive models further in this study.

The effects of predictors may take many periods to be felt and may not work their way through the economy for many periods still. A change in independent variable x_t may affect multiple subsequent values of the outcome $y_t, y_{t+1}, \ldots$ This can be accounted for in an infinite distributed lag probit model

$$\pi_t = \alpha + \sum_{k=0}^{\infty} \beta_k x_{t-k}$$

wherein all previous lags of the predictor variable are included in the specification, or a finite distributed lag model

$$\pi_t = \alpha + \sum_{k=0}^{q} \beta_k x_{t-k} \ .$$

The general form for the latent variable specification in the dynamic finite distributed lag model is thus

$$\pi_t = \alpha + \sum_{k=0}^{q_1} \beta_{1,k} x_{1,t-h-k} + \sum_{k=0}^{q_2} \beta_{2,k} x_{2,t-h-k} + \ldots + \sum_{k=0}^{q_n} \beta_{n,k} x_{n,t-h-k} + \sum_{k=0}^{q_{n+1}} \gamma_k y_{t-h-k} \ .$$

The coefficients of the various lags give both the dynamic marginal effects and the cumulative effects of the explanatory variable on the outcomes over time as well as how patterns in the lags effect subsequent values of the outcome variable.

Once a model specification is decided and the parameters estimated, the fit and explanatory power of the model is measured in a variety of ways. The McFadden Pseudo R^2 (hereafter simply pseudo R^2) corresponds to the coefficient of determination in standard linear regression and likewise reflects the degree to which a model explains the variation in the response variable.

$$\text{pseudo } R^2 = 1 - \frac{\log L_u}{\log L_r},$$

where L_u is the likelihood function for the unrestricted model and L_r that for the restricted model.

For model comparison and model selection, the Akaike Information Criterion (AIC) and the Schwarz, or Bayesian, Information Criterion (BIC) are used, where smaller values reflect more efficient models in the sense of providing greater explanatory power but not at the expense of parsimony of explanatory variables.

$$\text{BIC} = \frac{2}{T}\left(k \log T - 2\log L\right),$$

$$\text{AIC} = \frac{2}{T}\left(k - \log L\right),$$

where k is the number of parameters to be estimated, T the number of in-sample observations, and L is the likelihood as a function of the estimated parameters. Note that the effect of k in the expressions penalizes for additional parameters.

For measures of out-of-sample fit, I use the root-mean-square-error (RMSE), mean-absolute-error (MAE), and Theil inequality coefficient as well as the percent of correct predictions, of true positives, of false positives, of true negatives and of false negatives.

$$\text{RMSE} = \sqrt{\frac{1}{r}\sum_{t=T+1}^{T+\tau}(p_t - y_t)^2} \; ,$$

$$\text{MAE} = \frac{1}{r}\sum_{t=T+1}^{T+\tau}\left|p_t - y_t\right|,$$

and

$$\text{Theil inequality coefficient} = \frac{\sqrt{\sum_{t=T+1}^{T+\tau}(p_t - y_t)^2}}{\sum_{t=T+1}^{T+\tau} y_t^{\,2}} \; ,$$

where *r* is the number of out-of-sample periods.

Results

For the forecast horizon I chose a lags of two quarters, since that would be a minimum amount of time for corrective monetary or fiscal policy to start taking effect. Static models with single variables and their various lags were considered first. For a two quarter ahead forecast for variables spread and CAPEd1 I started with lag -2 alone and successively added lags -3, -4 and so forth while using the AIC, BIC and the significance of the last lag coefficient to decide how many lags of each variable to use. Likewise, for CSd1, but starting at lag -3 since there is a two-month delay in the release of the Case-Shiller index. The increase in the pseudo R^2 as lags are added was also considered in selecting the number of lags. If the increase in the R^2 is not very large compared to previous increases, this suggests that little explanatory power is added by the additional lag. Finally, the joint significance of a variable and its lags in the model combining the variables is considered as well to check for the overall significance of a variable and its lags.

For the variable spread, lags -2 to -5 were chosen on the basis of the lowest AIC and BIC and the longest sequence of lags with the last lag's coefficient being significant and for the fact

Table 2 Lag determination criteria

The tables are for models with a single variable and its lags, except for the significance of the variable's lags in the complete model with all three variables (SP500 excluded) with the selected lags.

The full static model includes spread lags -2 to -5, CAPE d1 lags -2 to -4, and CS d1 lag -3, in sample 1955q1 through 1999q4.

Variable: spread

	Last lag (starting at -2)				
	-2	-3	-4	-5	-6
AIC	0.665	0.616	0.611	0.588	0.600
BIC	0.700	0.670	0.683	0.679	0.709
Pseudo R^2	0.222	0.302	0.324	0.368	0.370
p-value for last lag coefficient	0.000	0.001	0.067	0.013	0.508
p-value for model				0.0000	
p-value for the joint significance of the spread lags in the full static model				0.0001	

Lags through -5 were select.

Variable: CAPE d1

	Last lag (starting at -2)				
	-2	-3	-4	-5	-6
AIC	0.715	0.687	0.670	0.678	
BIC	0.750	0.740	0.741	0.767	
Pseudo R^2	0.161	0.209	0.243	0.246	
p-value for last lag coefficient	0.000	0.009	0.026	0.479	
p-value for model			0.0000		
p-value for the joint significance of the CAPE d1 lags in the full static model			0.0249		

Lags through -4 were select.

Variable: SP500 percent change

	Last lag (starting at -2)		
	-2	-3	-4
AIC	0.753	0.755	0.776
BIC	0.718	0.702	0.704
Pseudo R^2	0.164	0.200	0.214
p-value for last lag coefficient	0.000	0.023	0.140
p-value for model		0.0000	

Lags through -3 were select for comparison to CAPE d1.

Variable: CS d1

	Last lag (starting at -3)		
	-2	-3	-4
AIC		0.784	0.786
BIC		0.820	0.839
Pseudo R^2		0.078	0.089
p-value for last lag coefficient		0.001	0.198
p-value for model		0.0007	0.0014
		0.1128	

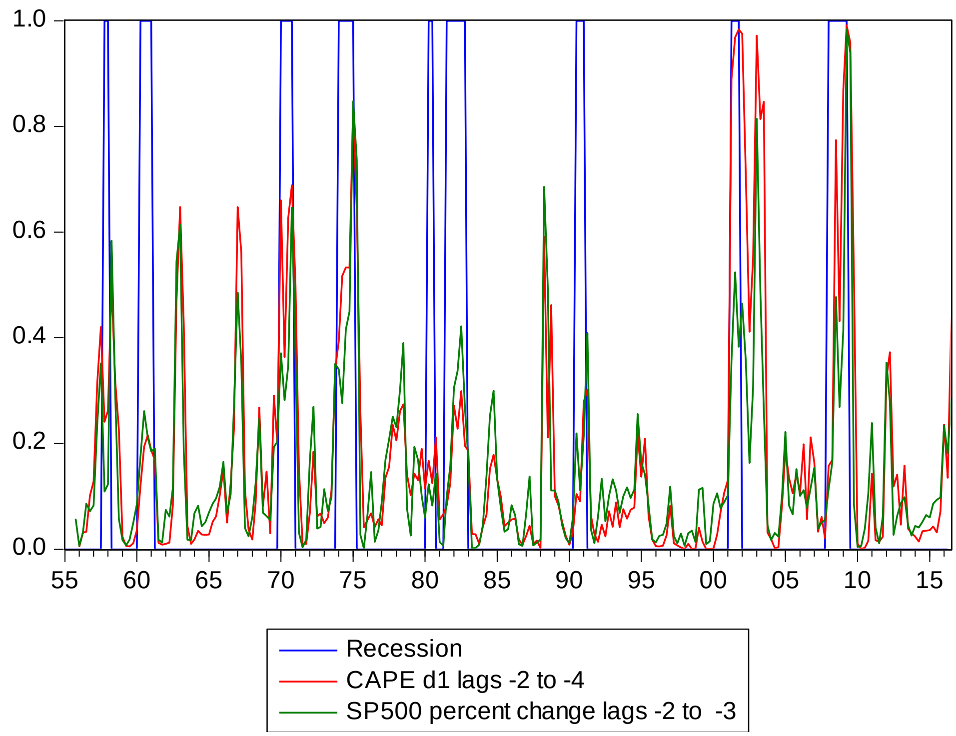
Lags through -3 were select.

that adding another lag barely increases the pseudo R^2. For the variable CAPEd1, lags -2 to -4 were chosen on the basis of the lowest AIC, the very nearly the lowest BIC (no lag combination has both lowest AIC and BIC) and longest lag sequence with the last lag's coefficient being significant. Also, an additional lag adds almost nothing to the Pseudo R^2. Similar criteria were used to select lag -3 alone for the CSd1 variable (see table 1). The percent change in the S&P 500 was similarly considered to compare its performance to the CAPE.

Figure 9, below, gives the probability of a recession state based on models with the CAPE first difference and the percent change in the S&P 500 and their lags as the sole predictor variables. A visual comparison of the predictive power of the S&P 500 variable and the CAPEd1 shows that the CAPEd1 produces a dramatically stronger and better timed forecast of the 2001 recession. Even for the in-sample, the pseudo R^2 and the information criteria show the CAPEd1 and its optimal lags produce a better model than the S&P 500 percent change and its best lags. Hence the CAPE has advantages that lead me to use it rather than the S&P 500 index as in for example Ng 2011. Note that a strong, well timed signal of the onset of a recession is more useful planners and central bankers than signal of the conclusion of a recession since the effect of stimulatory policy extending beyond the conclusion of a recession seldom is as much cause for much concern as a delay initiating stimulatory policy in advance of the onset of a recession.

Figure 9

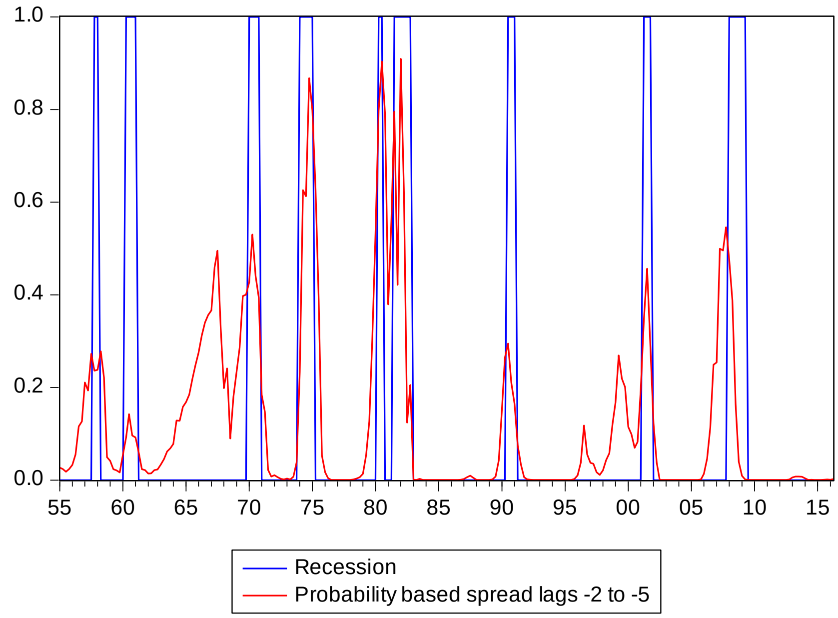
Recession probabilities based on the CAPE and based on the S&P 500



Graphs of the probabilities of recession based solely on single variables and their lags show that each contains valuable information on the timing of recession and furthermore suggests that each variable contains information the others do not. The graph for the probability computed using only spread (Figure 10) shows the information in the spread alone has predictive power in signaling the beginnings of both the 2001 and 2008 recessions though the signals are rather weak. The graph using the CAPEd1 variable alone (Figure 11) is remarkable for its pronounced signals for both of the out-of-sample recessions but especially for the 2001 recession. This is what one might expect given that that recession was associated with a stock bubble. It predicts the onset of the 2001 recession precisely and with a very strong signal while its timing on the 2008 recession is late. The CSd1 based graph (Figure 12) is remarkable as well. While it fails to signal the 2001 recession it gives both a well-timed and very strong signal of the 2008 recession. Again, this is as we might expect for reasons analogous to those for the 2001 recession being forecasted by stocks prices, the 2008 recession was signaled by home prices. The CAPE and CS variables give complementary information. Each of the immediately following graphs are for the estimated model given bellow the graph.

Figure 10

Recession probabilities based on the spread

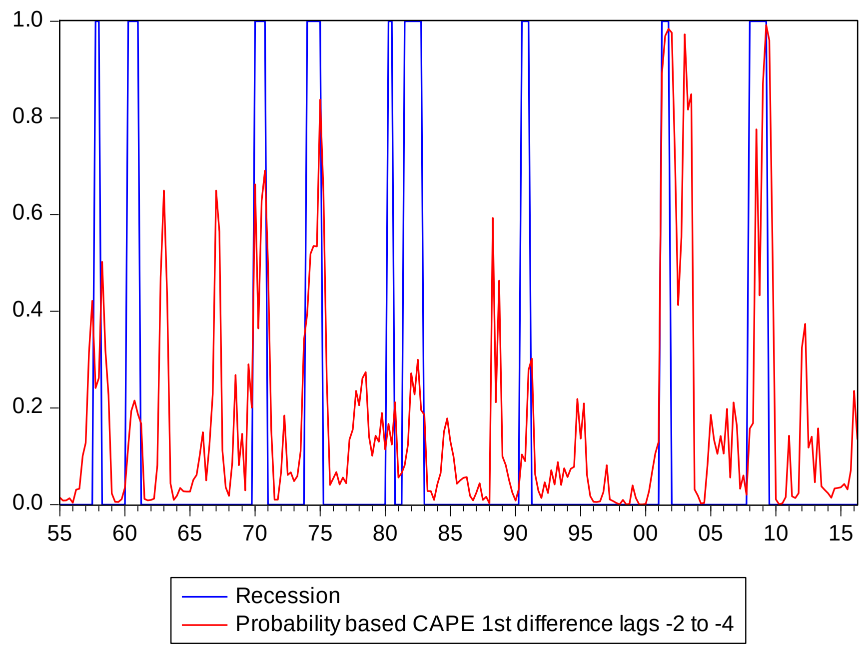


$$P_{t-2}(y_t = 1) = E_{t-2}(y_t) = \Phi(\alpha + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4)$$

Note the weaker signal of the two recent recessions but fewer false positive spikes than the model based on asset variables alone.

Figure 11

Recession probabilities based on the CAPE

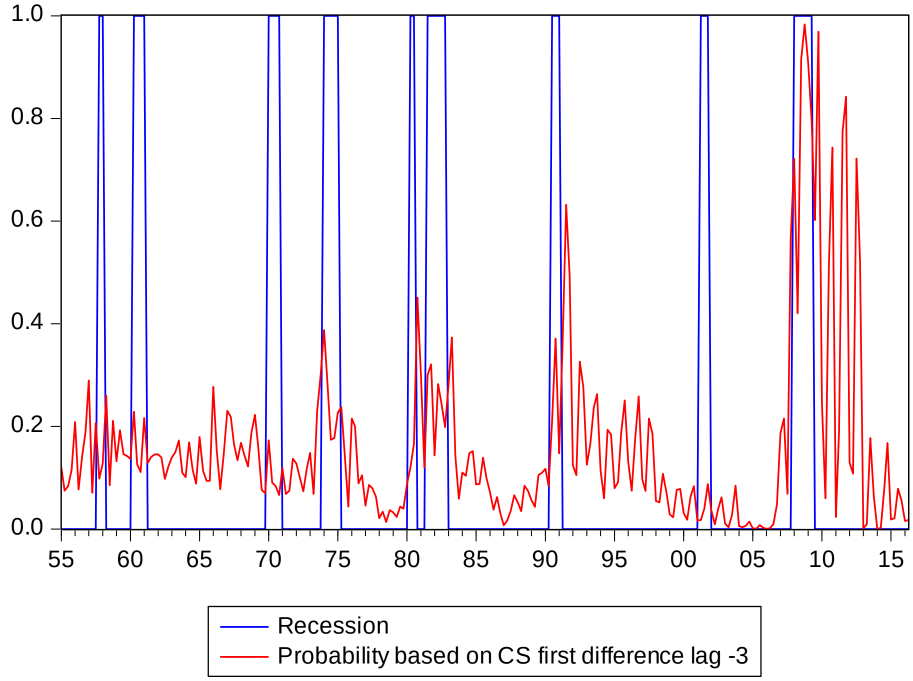


$$P_{t-2}(y_t = 1) = E_{t-2}(y_t) = \Phi(\alpha + CAPEd1_{t-2}\gamma_1 + CAPEd1_{t-3}\gamma_2 + CAPEd1_{t-4}\gamma_3)$$

Note the strong and timely signal for the 2001 recession. The 2008 is forecasted as well but late.

Figure 12

Recession probabilities based on the CS

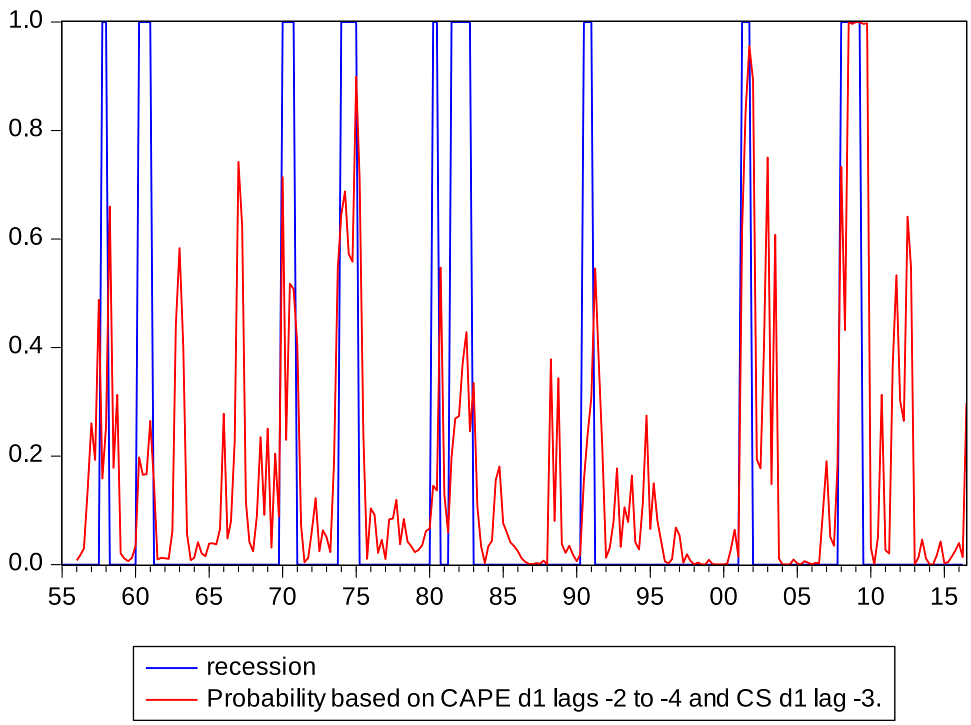


$$P_{t-2}(y_t = 1) = E_{t-2}(y_t) = \Phi(\alpha + CSd1_{t-3}\delta_1)$$

Note the failure to signal the 2001 recession but the strong signal and moderately good timing for the 2008 recession and several false positive peaks following.

Figure 13

Recession probabilities based on the CAPE and the CS jointly



$$E_{t-2}(y_t) = \Phi(\alpha + CAPEd1_{t-2}\gamma_1 + CAPEd1_{t-3}\gamma_2 + CAPEd1_{t-4}\gamma_3 + CSd1_{t-3}\delta_1)$$

The predictive power of the two asset variables together, static model (dynamic model, not shown, is very similar).

The model with the combined asset variables and their lags

$$E_{t-2}(y_t) = \Phi(\alpha + CAPEd1_{t-2}\gamma_1 + CAPEd1_{t-3}\gamma_2 + CAPEd1_{t-4}\gamma_3 + CSd1_{t-3}\delta_1) \quad \textit{Equation 1}$$

has a lager pseudo R^2 than the one above for the spread lags alone and outperforms it on every measure of out-of-sample fit except false positives/true negatives and the MAE. The response to the asset variables for the out-of-sample recessions (Figure 13) is visually striking in the timing and the strength of the signal, but does give considerably more false positives than the spread variable and its lags give. More useful and actionable information is given by the asset variables than the spread variable. and its lags. Table 2 below compares the performance measures of the model with spread lags alone to that with only the asset variable lags alone.

Table 3

In sample factors	spread alone, lags -2 to -5	CAPE d1 lags -2 to -4 and CS d1 lag -3
AIC	0.589	0.648
BIC	0.679	0.738
Pseudo R^2	0.368	0.294
p-value for last lag coefficient of the lags	0.013	0.027 and 0.005
Out of sample factors		
RMSE	0.314	0.302
MAE	0.149	0.159
Theil Coef	0.584	0.378
Out-of-sample at 50% threshold. 9 recession and 57 non-recession quarters.		
false positive	1	6
false negative	9	1
true positve	0	8
true negative	56	51

Distributed lags not only describe the persisting effect of changes in predictor variables, but also capture information on how the relationship among the lags effects the dependent variable. I use an unstructured distributed lag, not forcing any pre-specified relationship between the lag coefficients. The collinearity of the lags, however, makes for very large standard errors for the coefficient estimate and hence very large *p*-values for the individual lags. Our concern however, is with the joint significance of the lags, the fit of the in-sample forecast, and most importa

ntly, the predictive power in the out-of-sample forecast. As it turns out, the model fares very well on all of these accounts. Information is captured by the model despite the uncertainty of in estimates of the individual coefficients and that the predictive power of the model is stark and impressive despite the very wide confidence intervals for the coefficient. The excellent fit of the model both in and out-of-sample gives me confidence in the model as a whole. The additional lags, however, add little to the predictive power and accuracy of the model out-of-sample. Thus I adopt the following static and dynamic specifications which I will refer to as the "full" or "complete" models.

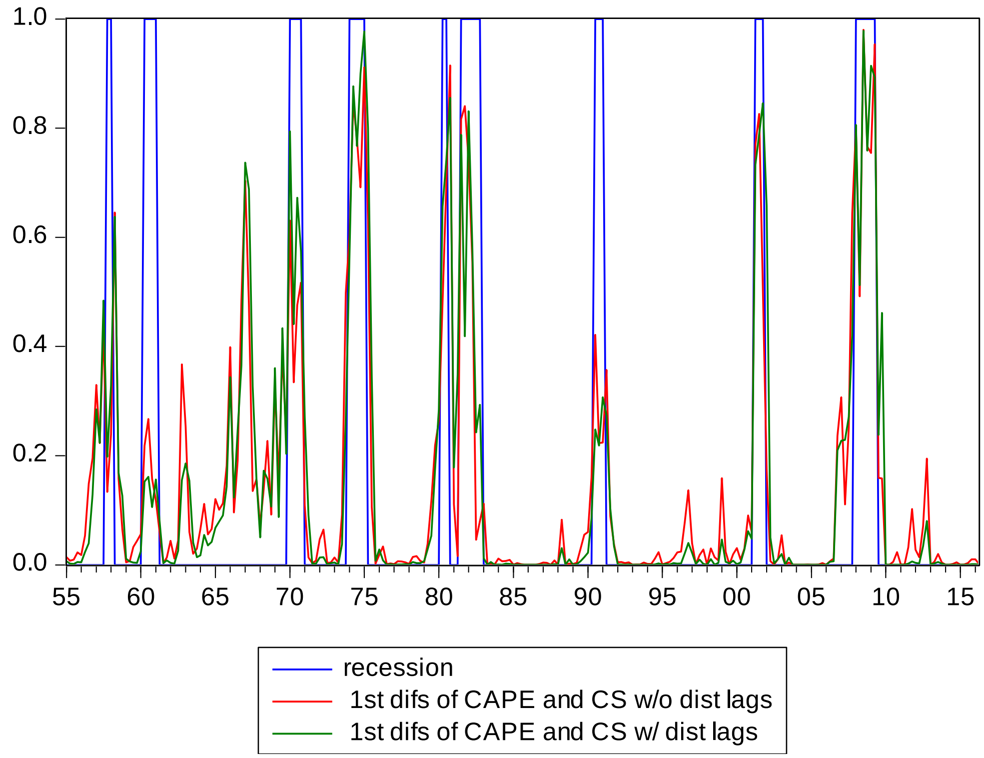
$$\begin{aligned} E_{t-2}(y_t) = \Phi(\alpha + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4 \\ + CAPE_{t-2}\gamma_1 + CAPE_{t-3}\gamma_2 + CAPE_{t-4}\gamma_3 + CS_{t-3}\delta_1) \end{aligned} \qquad \textit{Equation 2}$$

$$\begin{aligned} E_{t-2}(y_t) = \Phi(\alpha + y_{t-2}\mu + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4 \\ + CAPE_{t-2}\gamma_1 + CAPE_{t-3}\gamma_2 + CAPE_{t-4}\gamma_3 + CS_{t-3}\delta_1) \end{aligned} \qquad \textit{Equation 3}$$

Once the model with the combination of the variables and their lags was estimated, each variable and its lags were tested for joint significance. Only the significance of the lag of CSd1 was poor with a p-value of 0.11 (Table 1 above). A comparison of the graphs of the restricted and unrestricted model (Figure 15) for in-sample show very little difference. However, with the benefit of foresight, estimating the model with an in-sample of 1955q1 through 2016q2 gives similar coefficient (-38.6) but a p-value of 0.001 for CSd1 lag. Furthermore, it is our objective to test the hypothesis that the 2008 recession would be forecast by a house price index. Thus for the combined reasons, I chose to keep CSd1. As it turns out, the unrestricted model does much better out-of-sample, especially so for the 2008 recession, again as we might expect.

Figure 14

Recession probabilities based on the spread, CS and CAPE and distributed lags versus without distributed lags



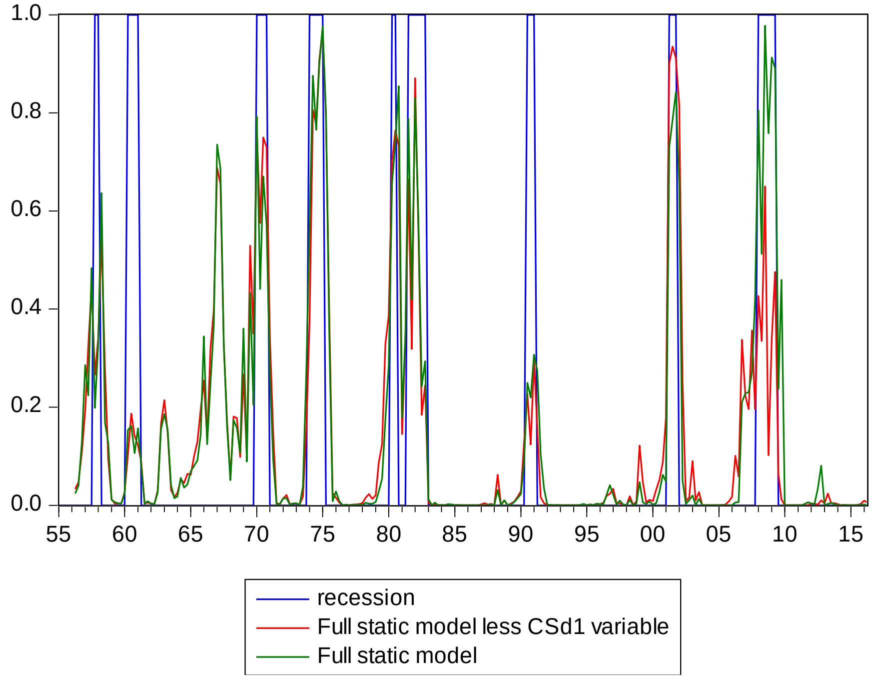
$$E_{t-2}(y_t) = \Phi(\alpha + spread_{t-3}\beta + CAPE_{t-2}\gamma + CS_{t-3}\delta)$$

$$E_{t-2}(y_t) = \Phi(\alpha + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4$$
$$+ CAPE_{t-2}\gamma_1 + CAPE_{t-3}\gamma_2 + CAPE_{t-4}\gamma_3 + CS_{t-3}\delta_1)$$

Distributed lags are found to add little to the predictive power of the model. The model without distributed lags incorporates the best single lags.

Figure 15

Recession probabilities based on complete model versus the complete model less the CSd1 variable

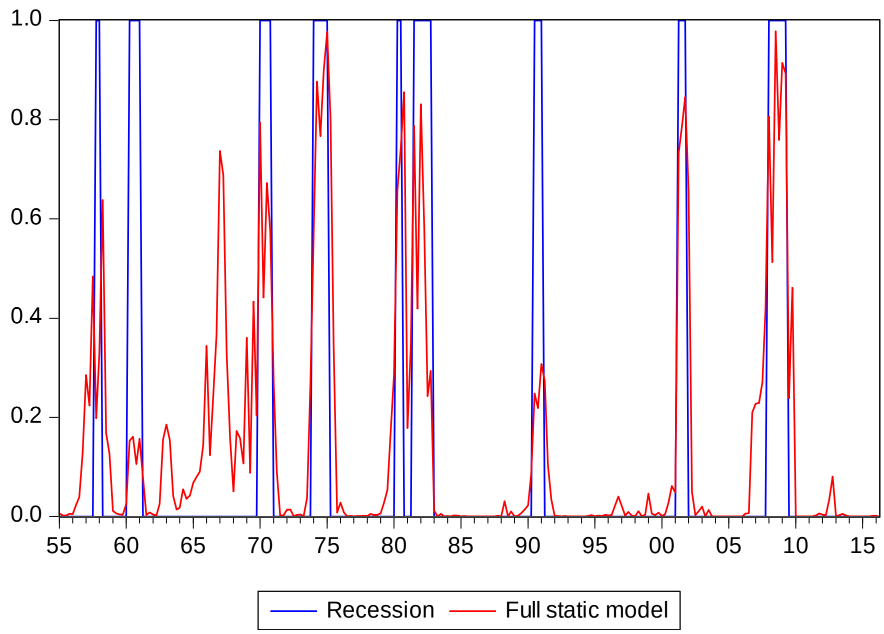


$$E_{t-2}(y_t) = \Phi(\alpha + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4$$
$$+ CAPE_{t-2}\gamma_1 + CAPE_{t-3}\gamma_2 + CAPE_{t-4}\gamma_3 + CS_{t-3}\delta_1)$$

The CSd1 enhances the out-of-sample fit considerably, particularly with respect to strong, well-timed forecasts of the onset of the 2008 recession.

Figure 16

Recession probabilities based complete model



$$E_{t-2}(y_t) = \Phi(\alpha + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4 + CAPE_{t-2}\gamma_1 + CAPE_{t-3}\gamma_2 + CAPE_{t-4}\gamma_3 + CS_{t-2}\delta_1 + CS_{t-3}\delta_2)$$

Note the strong false positive in 1967 corresponding to a quarter with < 0.1% growth.

While it is not realistic to assume the forecaster will know the recession state of the economy in real-time, for forecasting the onset of recession, one might assume that the current quarter is in non-recession. Typically, after one or two quarters into a recession, it is apparent that the general economy is in decline. We will assume for the dynamic model, unrealistically, that we know the state of the economy in the quarter from which the forecast is made, that is, two quarters in advance of the quarter to be forecasted. If even this model does not produce substantial improvements of the static model, then it seems unlikely that a dynamic model, using realistically obtainable data, would produce much of an improvement. More sophisticated models, such as a dynamic autoregressive models and ones using iterated forecasting give marginal improvements (cf. Kauppi and Saikkonen, 2008; Ng, 2011).

The dynamic model

$$E_{t-2}(y_t) = \Phi(\alpha + \mu y_{t-2} + spread_{t-2}\beta_1 + spread_{t-3}\beta_2 + spread_{t-4}\beta_3 + spread_{t-5}\beta_4 + CAPE_{t-2}\gamma_1 + CAPE_{t-3}\gamma_2 + CAPE_{t-4}\gamma_3 + CS_{t-3}\delta_1) \quad \textit{Equation 4}$$

was estimated and compared to the static model. The static model does better on every criteria of both in and out-of-sample fit, except of course, the pseudo R^2.

What improvements to the model would we expect by adding lags of the outcome variable? If a give quarter is not a recession quarter, the probability of the following quarter not being a recession is quite high given that most quarters are not recessions quarters. If the current quarter is a recession quarter, the following quarter is now much more likely to still be a recession. Estimating the model $E_{t-2}(y_t) = \Phi(\alpha + \mu y_{t-2})$, as seen from the graph below (Figure 18) of the probability, no practical predictive power is given by this simple dynamic model. If the current quarter is a recession quarter, it gives the probability of two quarters ahead as being 0.46 and if the current quarter is not a recession then it gives the probability two quarters ahead as being 0.09. Hence, we would not expect much of an improvement of the predictive power of the dynamic over the static model. The lagged dependent variable may give us a better idea of the duration of a recession but no useful information as to the onset of a recession. This is consistent with the findings of Ng (2011).

Table 4 Comparison of the in-sample and out-of-sample performance of the models

In-sample 1955q1 through 1999q4

Factors	Static model	Standard errors*	Without distributed lags Static model	Standard errors	Dynamic model	Standard errors*
constant	-0.464	0.254	-0.609	0.205	-0.620	0.277
spread t-2	-0.211	0.248			-0.430	0.292
spread t-3	-0.527	0.363	-0.720	0.174	-0.470	0.370
spread t-4	0.232	0.364			0.242	0.381
spread t-5	-0.490	0.273			-0.328	0.299
CAPE d1 t-2	-8.912	3.710	-9.472	3.177	-7.447	3.850
CAPE d1 t-3	-0.480	3.807			0.376	3.899
CAPEd1 t-4	-5.550	3.996			-4.380	4.184
CS d1 -3	-34.043	21.702	-36.865	19.399	-29.761	21.676
y_{t-2}					0.773	1.513
AIC	0.562		0.549		0.560	
BIC	0.725		0.620		0.741	
Pseudo R^2	0.453		0.397		0.469	

% correct in-sample

Out of sample 2000q1 to 2016q2

RMSE	0.156	0.150	0.173
MAE	0.074	0.075	0.083
Theil Ineq Coef	0.224	0.229	0.260

Out-of-sample at 50% threshold.
9 recession and 57 non-recession quarters.

false positive	1	1	1
false negative	0	1	1
true positve	9	8	8
true negative	56	56	56
% correct	98.4	97.0	97.0

*See discussion on large standard errors

Figure 17

Probabilities based on the complete static and complete dynamic models

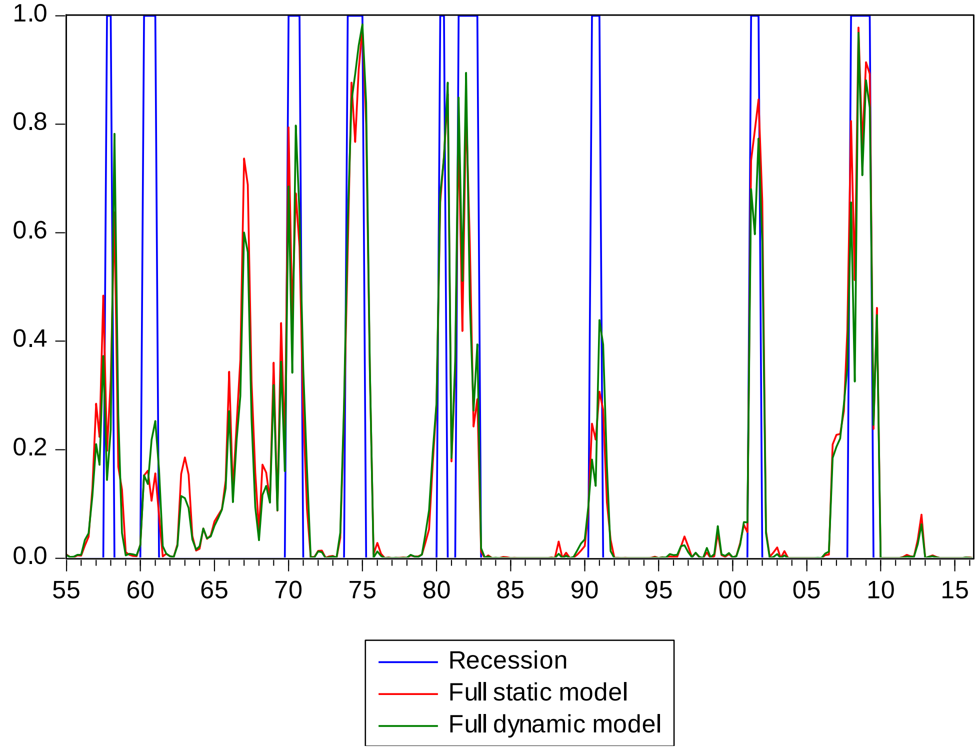
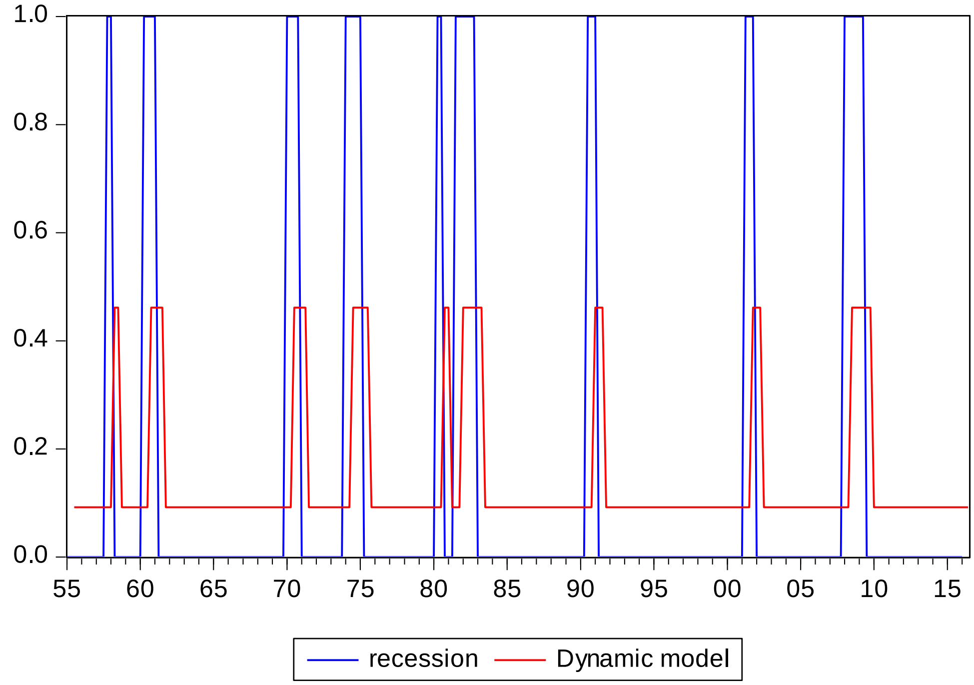


Figure 18

The simple dynamic model gives no practical predictive power as to the onset of recessions.



$$E_{t-2}(y_t) = \Phi(\alpha + \mu y_{t-2})$$

Table 4 below gives the forecasted recession probabilities in the quarters leading up to the two out-of-sample recessions. Recession quarters highlighted. The tables in the third column give the estimated probabilities with the S&P 500 variable substituted for the CAPE variable. The tables in the last column show the forecasted probabilities with the model parameters re-estimated each successive quarter based on actual data from preceding quarters as so using all available data at the time of the prediction.

Table 4 Forecasted recession probabilities.

Dynamic model		
Quarter	Probability	Actual
2000Q2	0.0037	0
2000Q3	0.0246	0
2000Q4	0.0664	0
2001Q1	0.0662	0
2001Q2	0.6801	1
2001Q3	0.5969	1
2001Q4	0.7735	1
2002Q1	0.5834	0
2002Q2	0.0459	0
2002Q3	0.0004	0
2002Q4	0.0022	0

Static model		
Quarter	Probability	Actual
2000Q2	0.0040	0
2000Q3	0.0275	0
2000Q4	0.0614	0
2001Q1	0.0480	0
2001Q2	0.7329	1
2001Q3	0.7886	1
2001Q4	0.8455	1
2002Q1	0.6579	0
2002Q2	0.0497	0
2002Q3	0.0027	0
2002Q4	0.0105	0

Static model (S&P500)		
Quarter	Probability	Actual
2000Q2	0.0229	0
2000Q3	0.0269	0
2000Q4	0.0473	0
2001Q1	0.0531	0
2001Q2	0.3226	1
2001Q3	0.5343	1
2001Q4	0.3366	1
2002Q1	0.1912	0
2002Q2	0.0226	0
2002Q3	0.0003	0
2002Q4	0.0013	0

Static model ongoing reestimation		
Quarter	Probability	Actual
2000Q2	0.0040	0
2000Q3	0.0278	0
2000Q4	0.0618	0
2001Q1	0.0471	0
2001Q2	0.7253	1
2001Q3	0.7778	1
2001Q4	0.8697	1
2002Q1	0.7370	0
2002Q2	0.0800	0
2002Q3	0.0012	0
2002Q4	0.0062	0

Dynamic model		
Quarter	Probability	Actual
2007Q1	0.2051	0
2007Q2	0.2205	0
2007Q3	0.2840	0
2007Q4	0.3521	0
2008Q1	0.6557	1
2008Q2	0.3254	1
2008Q3	0.9683	1
2008Q4	0.7057	1
2009Q1	0.8810	1
2009Q2	0.8303	1
2009Q3	0.2456	0
2009Q4	0.4481	0
2010Q1	0.0000	0
2010Q2	0.0000	0

Static model		
Quarter	Probability	Actual
2007Q1	0.2273	0
2007Q2	0.2289	0
2007Q3	0.2709	0
2007Q4	0.4195	0
2008Q1	0.8060	1
2008Q2	0.5124	1
2008Q3	0.9780	1
2008Q4	0.7589	1
2009Q1	0.9144	1
2009Q2	0.8929	1
2009Q3	0.2383	0
2009Q4	0.4613	0
2010Q1	0.0000	0
2010Q2	0.0000	0

Static model (S&P500)		
Quarter	Probability	Actual
2007Q1	0.2708	0
2007Q2	0.2662	0
2007Q3	0.2047	0
2007Q4	0.5568	0
2008Q1	0.7323	1
2008Q2	0.5319	1
2008Q3	0.8903	1
2008Q4	0.6430	1
2009Q1	0.5050	1
2009Q2	0.8582	1
2009Q3	0.2652	0
2009Q4	0.0351	0
2010Q1	0.0000	0
2010Q2	0.0000	0

Static model ongoing reestimation		
Quarter	Probability	Actual
2007Q1	0.2454	0
2007Q2	0.2250	0
2007Q3	0.2486	0
2007Q4	0.4794	0
2008Q1	0.8244	1
2008Q2	0.4922	1
2008Q3	0.7807	1
2008Q4	0.8661	1
2009Q1	0.9476	1
2009Q2	0.9745	1
2009Q3	0.3684	0
2009Q4	0.6575	0
2010Q1	0.0000	0
2010Q2	0.0000	0

The following graphs allow us to visually evaluated the robustness of the model to the change of in-sample periods, that last using the period 1990-2016 to retrodict recessions in the period 1955-1989.

Figure 19

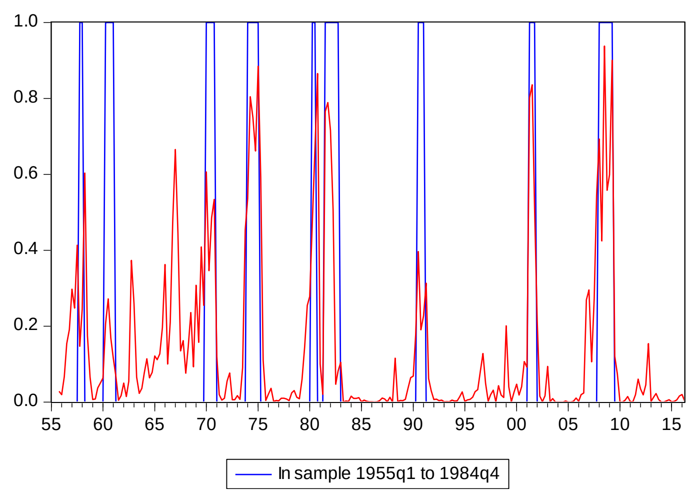


Figure 20

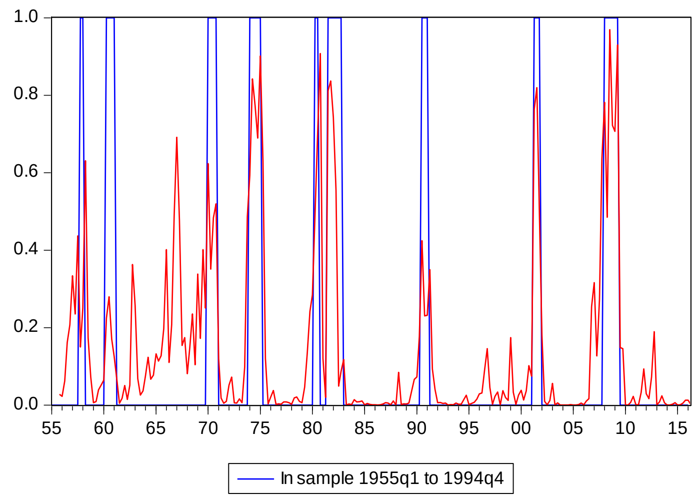
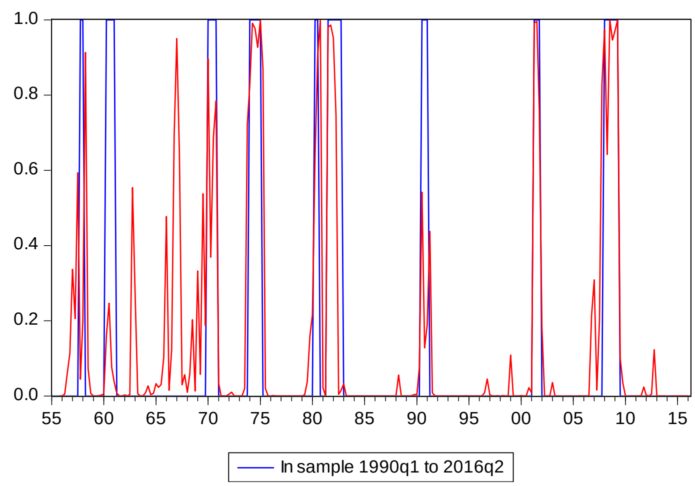
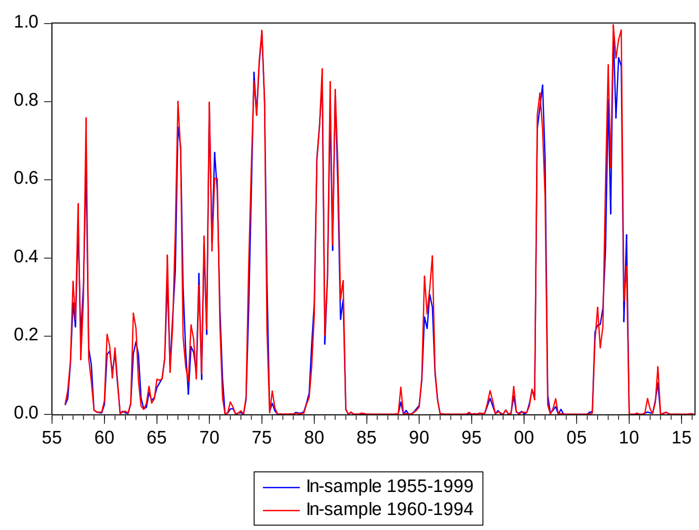


Figure 21



Below the graph compares the probabilities estimated with the in-sample period of 1960-1994 to that of an in-sample of 1955-1999. One can readily see the very slight differences reflecting the robustness of the model to changes to the in-sample periods.

Figure 22



Conclusion

To summarize, the major contributions and findings of this study are:

1) While the yield spread has for decades been recognized as the benchmark predictor of recessions, the two asset variables together have predictive power, in terms of actionable information, that exceeds that of the yield curve for the in and out-of-sample of this study.

2) This is so even though the in-sample is for a period wherein the recessions were not so markedly characterized as being preceded by asset bubbles as the 2001 and 2008 recession were. The in-sample period consisting of the last four and a half decades of the 20^{th} century contains information on the relationship between the two asset index variables and the recession indicator that is captured by the estimated model and expressed in the two out-of-sample recessions. So while the last two recessions may have had particularly pronounced antecedent causes in historically large asset bubbles, the results here suggest they are not qualitatively different in this regard from those of the previous four decades.

3) The CAPE produces both a better in-sample fit and substantially superior out-of-sample forecasts than the standard indices as used in previous studies (cf. Ng, 2011; Fossati, 2012; Christiansen et al, 2013).

4) The complete model (Eq. 2) estimated here provides truly actionable information for central bankers and policy makers concerned with counter-cyclical interventions.

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