The Exchange Rate and Macroeconomic Determinants: Time-Varying Transitional Dynamics

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Abstract

In this paper, I consider modeling the effects of the macroeconomic determinants on the nominal exchange rate to be channeled through the transition probabilities in a Markovian process. The model posits that the deviation of the exchange rate from its fundamental value alters the market's belief in the probability of the process staying in certain regime next period. This paper further takes into account the ARCH effects of the volatility of the exchange rate. Empirical results generally confirm that fundamentals can affect the evolution of the dynamics of the exchange rate in a nonlinear way through the transition probabilities. In addition, I find that the volatility of the exchange rate is associated with significant ARCH effects which are subject to regime change.

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1. Introduction

The floating exchange rate in the post-Bretton Woods era appears to be disconnected from its underlying macroeconomic determinants most of the time. Empirical work has often failed to present evidence of stable relationship between nominal exchange rate movements and fundamental variables suggested by the exchange rate determination models. "Indeed, the explanatory power of these models is essentially zero," as Evans and Lyons (2002, p.170) assert. In addition, a plethora of empirical studies find that the nominal exchange rate is excessively volatile relative to the underlying macroeconomic variables during the recent floating period. Flood and Rose (1999), for example, show that there are no macro-fundamentals capable of explaining the dramatic volatility of the exchange rate.

In this paper, I consider modeling the effects of the macroeconomic determinants on the nominal exchange rate to be channeled through the transition probabilities in a Markovian process. Many researchers have sought ways to model the possible nonlinearities in the relationship between the exchange rate and macro-fundamentals.[1] Little work, nevertheless, has ever studied the transitional effects of the macroeconomic determinants on the exchange rate. In effect, allowing fundamentals to affect the transition probabilities in the Markovian process is intuitively attractive: the market responds to the updated news in the macro variables - deviation of the exchange rate from its fundamental value - and in turn alters the belief in the chance of the process staying in certain regime next period.

[1] See, for example, Taylor and Peel (2000), Taylor *et al* (2001), and Kilian and Taylor (2001) consider an exponential smooth transition autoregressive (ESTAR) model to capture the nature of nonlinear mean reversion in real exchange rates, Wu and Chen (2001) propose a nonlinear error-correction model allowing for time-varying coefficients, and Qi and Wu (2003) employ a neural network to study the nonlinear predictability of exchange rates.

My work further takes into account the ARCH effects of the volatility of the exchange rate. Autoregressive conditional heteroskedasticity and related effects have been repeatedly documented in exchange rates. Diebold (1988), for example, finds strong ARCH effects in all the seven nominal exchange rates examined. The ARCH (GARCH) models have been extensively applied to financial time series and have probably become one the most popular tools to study financial market volatility since the pioneering works by Engle (1982), Bolleslev (1986), and Bollerslev and Engle (1986). The application of ARCH models, however, may be problematic according to Lamoureux and Lastrapes (1990) since ARCH estimates are seriously affected by structural changes or regime shifts. On the other hand, the Markov-switching model popularized by Hamilton (1988, 1989) has proved especially successful in modeling time series with changes of regime.[2] Nevertheless, the Hamilton's Markov-switching model takes little consideration of the movements in the variance. For example, Pagan and Schwert (1990) show that Markov-switching specification is not satisfactory in explaining the monthly U.S. stock-return volatility from 1834 to 1925. In this regard, an extension combining the traditional Markov-switching model with ARCH specification turns out to be a natural motivation.

Using four major dollar exchange rates, I investigate the potential transitional effects of macroeconomic determinants and ARCH effects in the volatility of the exchange rate. A variety of fundamentals-based models are considered to measure the fundamental value of the exchange rate, including the purchasing power parity model, Mark's (1995) specification, the real interest differential model, and Hooper and Morton's (1982) portfolio balance model. Empirical results generally confirm that macroeconomic determinants can affect the evolution of the dynamics of

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[2] Prominent applications include, but not limit to, Hamilton's (1989) model of trends in the business cycle, Turner, Startz and Nelson's (1989) model of excess return and volatility, Cecchetti, Lee, and Mark's (1990) model of mean reversion in equilibrium asset prices, Engel and Hamilton's (1990) model of exchange rate dynamics, Raymond and Rich's (1997) model of the relationship between oil price shocks and macroeconomic fluctuations, and Psaradakis, Sola, and Spagnolo's (2004) model of stock prices and dividends.

the exchange rate in a nonlinear way through the transition probabilities. Results further reveal that the volatility of the exchange rate is associated with significant ARCH effects which are subject to regime change.

The remainder of the chapter is structured as follows. Section 2 specifies the time-varying Markov-switching ARCH model. Section 3 describes data, estimation and forecast procedure. Section 4 presents empirical results. Section 5 concludes.

2. Model Specification

2.1 The Markov-switching ARCH Model

I consider modeling the logarithm of dollar-priced exchange rate, e_t, in the context of Engle's (1982) autoregressive conditional heteroskedasticity (ARCH) specification and allow for regime-switching in the parameters. This framework facilitates capturing time-variant effect of the conditional variance and accounting for the possible parameter instability in exchange rate models due to changes in international monetary policies and global trade patterns, shocks to important commodity markets, and particularly rare events such as market crashes, financial panics, and economic turmoils. The two-state Markov-switching ARCH model can be characterized as follows:

$$y_t \equiv e_t - e_{t-1}$$

$$y_t = \mu_{s_t} + u_t$$

$$u_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1)$$

$$\sigma_t^2 = \alpha_{st} + \sum_{j=1}^{q} \beta_{j,s_t} u_{t-j}^2 \tag{1}$$

where $s_t \in \{1,2\}$ is the latent state variable governing the regime shifts in the data generating process of the exchange rate. A generalized ARCH (GARCH) specification introduced by Bollerslev (1986) allows for the conditional variance depending not only on the lagged squares of the disturbance term but also on its own lagged values:

$$\sigma_t^2 = \propto + \sum_{j=1}^{q} \beta_j \ u_{t-j}^2 + \sum_{j=1}^{q} \gamma_j \ \sigma_{t-j}^2 \tag{2}$$

The GARCH specification combined with regime-switching as in (2), however, is essentially intractable and extremely hard to estimate due to the fact that the conditional variance σ_t^2 is a function of the entire past history of the state variables.[3] Therefore, I herein focus on modeling the conditional variance in the exchange rate with low-order ARCH progress. The most remarkable merit of adopting ARCH relative to GARCH in the context of regime-switching scenario is to ease the problem of path dependence of the conditional variance and thus make it computationally tractable, without losing the effectiveness of accounting for time-varying conditional variance structure.

Cai (1994) and Hamilton and Susmel (1994) apply the Markov-switching ARCH model respectively to study treasury bill yields and stock market returns. They both find the model perform very well in capturing regime shifts in the financial time series and time-variant conditional second moments within the regimes. The present analysis adopts a similar framework as in these previous studies but differs in two important aspects. First, I allow for the ARCH effects state-dependent both in the intercept and the coefficients on the lagged conditional variance. In contrast, Cai (1994) allows for regime-switching only in the intercept component while Hamilton and Susmel (1994) scale up the ARCH process by different fixed constants

[3] Recent studies by Gray (1996) and Klaassen (2002) have attempted to make improvement over the regime-switching GARCH. Nevertheless, its analytical intractability remains a serious drawback (see Haas, Mitnik, and Paolella 2004).

according to different states, maintaining the intercept and coefficients both state-independent. Second, instead of imposing a fixed transition probability matrix on regime shifts, I consider a different transitional evolution of the exchange rate dynamics where transition probabilities change over time as described below.

2.2. Time-Varying Transition Probabilities

The unobserved state variable s_t is assumed to follow first-order Markov process with time-varying transition probabilities

$$P(s_t = j|s_{t-1} = i) = p_t^{ij}(z_{t-1}) \tag{3}$$

where $p_t^{ij}(\cdot)$ is a function of a $(k \times 1)$ vector of observed exogenous or predetermined variables z_{t-1}, and $\sum_j p_t^{ij} = 1 \; \forall i, j \in \{1,2\}$. In the present analysis, the observed information vector z_{t-1} contains a constant, the lag of change in exchange rates y_{t-1}, and d_{t-1}, the deviation of the spot exchange rate from its equilibrium level or fundamental value determined by the prevailing macroeconomic models described in the following subsection.

The transition probabilities are further assumed to be evolving as logistic functions of z_{t-1}. Specifically, the transition probability matrix is given as

$$P_t \equiv \begin{pmatrix} p_t^{11} & p_t^{12} \\ p_t^{21} & p_t^{22} \end{pmatrix}$$

with

$$P_t^{11} = \frac{\exp(a \cdot z_{t-1}^{'})}{1+\exp(a \cdot z_{t-1}^{'})}, \qquad P_t^{12} = \frac{1}{1+\exp(a \cdot z_{t-1}^{'})},$$

$$P_t^{21} = \frac{1}{1+\exp(b \cdot z_{t-1}^{'})}, \qquad P_t^{22} = \frac{\exp(b \cdot z_{t-1}^{'})}{1+\exp(b \cdot z_{t-1}^{'})}, \tag{4}$$

and

$$z_t = (1 \; y_t \; d_t)'$$

$$d_t = e_t - f_t$$

where f_t is the fundamental value of the exchange rate determined by macroeconomic determinants.

Diebold, Lee, and Weinbach (1994) provide a tractable methodology to derive the maximum likelihood estimation based on an EM algorithm. They demonstrate that their extension to Hamilton's Markov-switching model by allowing for time-varying transition probabilities not only nests the framework with fixed transition probabilities but also better describes the true data generate process through simulation.[4]

2.3. Macroeconomic Determinants

The standard monetary approach has suggested a host of leading models regarding exchange rate determination. To evaluate different transitional effects of the macroeconomic determinants, I select four comparators in line with previous prominent studies, which include the purchasing power parity (PPP), Mark's (1995) specification, Frankel's (1979) the real interest rate differential (RID) model, and the portfolio balance model according to Hooper and Morton (1982).

2.3.1. PPP

One building block of the monetary models is the purchasing power parity (PPP), which defines the exchange rate as the relative price of two monies. The underlying rationale of the PPP hypothesis is that goods-market arbitrage tends to move the exchange rate to equalize prices in two countries. A PPP fundamental is thus given based on relative consumer price indices

$$f_t = p_t - p_t^* \tag{5}$$

[4] Empirical applications of the time-varying transition probability Markov-switching model can also be seen in variety of settings. Filardo (1994), for example, models transition probabilities as a function of leading indicator variables, Durland and McCurdy (1994) modeled transition probabilities as duration-dependent, and Ghysels (1994) considered periodicity and seasonality in the Markov process.

where f_t denote the fundamental value determined by macroeconomic variables, p_t is the logarithm of the domestic price level and p_t^* is the logarithm of the foreign price level.[5] And the deviation of the nominal exchange rate from the underlying PPP fundamental is defined as

$$d_t = e_t - f_t \tag{6}$$

which in fact is the logarithm of the real exchange rate.

Testing the hypothesis of purchasing power parity can be dating back as early as a century ago and there is an expanding empirical literature with competing evidence.[6] Notably, Rogoff (1996) raises the purchasing power parity puzzle: no standard theory can explain the fact that the exchange rate follows an extremely slow adjustment toward the purchasing power parity while exhibits enormous short-run volatility. Taylor and Taylor (2004) have offered an excellent survey on this topic and concluded "… that long-run PPP may hold in the sense that there is significant mean reversion of the real exchange rate". This provides important insight for this study in that the deviation of the exchange rate from the PPP value may affect market participants' belief in next period's staying probability, which drives the exchange rate, albeit slowly, back toward its fundamental value in a nonlinear way.

2.3.2. Mark's Specification

Mark (1995) presents a monetary model which is one of the most prominent and striking examples of evidence in favor of long-horizon exchange rate predictability. He defines the fundamental value of the exchange rate as a linear combination of relative money and relative output

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[5] A commonly used proxy for price level is CPI. When calculating PPP exchange rate, the formula is slightly modified as: $f_t = f_t + cpi_t - cpi_t^*$, where f_o represents the PPP exchange rate that prevails in the base year between the two countries. Note that in order for this formula to work correctly, the CPIs in both countries must share the same base year.

[6] See, for example, Frenkel (1976, 1981), Frankel (1986), Editson (1987), Glen (1992), Taylor (1995), Frankel and Rose (1995), and Taylor and Chowdhury (2004).

$$f_t = (m_t - m_t^*) - \emptyset(q_t - q_t^*) \tag{7}$$

where m_t and q_t denote the log-levels of the domestic money supply and income at time t, $\emptyset$ is a constant, and asterisks denote foreign variables. Mark (1995) assumes that $\emptyset = 1$. Similarly, the deviation of the exchange rate from the fundamental value defined in Eq. (7) is given as (6).

According to Mark and Sul (2001), equation (7) is a generic representation of the long-run equilibrium exchange rate implied by modern theories of exchange rate determination, which is consistent with both traditional monetary models based on aggregate functions by Frenkel 1976), Mussa (1976), and Frenkel and Johnson (1978), and with more recent microfounded open economy models by Obstfeld and Rogoff (1995, 2000) and Lane (2001). Although nearly no empirical significance of the contemporaneous link between monetary fundamentals and the exchange rate has been established, Mark (1995) shows that the deviation of the nominal exchange rate from Eq. (7), $f_t - e_t$, has significant predictive power in determining the future change in exchange rate in 3-5 years. Some other researchers, such as Groen (2000), Mark and Sul (2001), and Rapach and Wohar (2002), on the basis of panel studies, have recently documented that the fundamentals described by Eq. (7) comove in the long run with the nominal exchange rate and therefore determine its equilibrium level.

***2.3.3.* RID**

Frankel (1979) presents another influential work regarding the relationship between monetary fundamentals and the exchange rate, which is usually referred to as the real interest rate differential (RID) model. The fundamental value of the exchange rate from the RID is given as

$$f_t = a_0 + a_1(m_t - m_t^*) + a_2(q_t - q_t^*) + a_3\left(i_t^s - i_t^{s^*}\right) + a_4(i_t^l - i_t^{l*}) \tag{8}$$

where i_t^s is the short-term interest rate and i_t^l is the long-term interest rate.

The RID extends the traditional flexible-price monetary model (see Frenkel 1976; Bilson 1978; and Hodrick 1978) by differentiating the impact on the exchange rate of short- and long-term interest rates. Particularly, the short-term interest rates are designed to capture liquidity or real effects of monetary policy while the long-term interest rates are designed to capture expected inflation effects. From a standard monetary perspective one would expect the coefficients on the relative money supply and long-term interest rates are positive (i.e. home currency depreciates) while the coefficients on the relative income and short-term interest rates are negative (i.e. home currency appreciates).

Frankel (1979) and MacDonald (1988) find supportive empirical evidence of the significant link between the exchange rate and the predicted fundamental value from Eq. (8) for the early part of the post Bretton Woods period. Numerous other researchers, however, show that the model does not perform well in the period beyond 1978, with estimated coefficients often wrongly signed or insignificant, and having poor in-sample performance (see Meese and Rogoff 1983; MacDonald and Taylor 1991; MacDonald 2004). The failure to establish the validity of the RID model for the exchange rate may be plausibly attributed to methodologically misuse the two-step cointegration method proposed by Engle and Granger (1987) according to MacDonald and Taylor (1994). They thus propose an appropriate multivariate estimation technique to obtain the cointegration vector of monetary variables in Eq. (8) and demonstrate that there are up to three statistically significant cointegrating vectors between the exchange rate and the relative money supplies, outputs, and long-term interest rates. Remarkably, their novel approach is shown to perform well in terms of both in-sample and out-of-sample criteria, with a robust outperformance over random walk at all five forecasting horizons examined. More recently, Frömmel, MacDonald and Menkhoff (2003) adopt a regime switching approach in which they allow the influence of monetary fundamental variables on the exchange rate to change over time, i.e., the RID model works for some periods but does not for the other periods. Their finding supports the view of a highly nonlinear and complex relationship between fundamentals and the exchange rate. In this regard, the present study is in the same line with their approach regarding the nonlinear relationship between macroeconomic determinants and the exchange rate while instead of assuming a regime switching in the influence (coefficients) of the monetary variables, I model the effects of macroeconomic determinants on the exchange rate to be channeled through the transition probabilities.

2.3.4. Portfolio Balance Model

The fourth fundamentals-based structural model is the portfolio balance model proposed by Hooper and Morton (1982). The fundamental value of the exchange rate according to the Hooper-Morton model can be expressed as a quasi-reduced form specification:

$$f_t = b_0 + b_1(m_t - m_t^*) + b_2(q_t - q_t^*) + b_3\left(i_t^s - i_t^{s^*}\right)$$
$$+b_4\left(\pi_t^e - \pi_t^{e^*}\right) + b_5(\overline{TB} - \overline{TB}^*) \tag{9}$$

where $\left(\pi_t^e - \pi_t^{e^*}\right)$ is the long-term expected inflation differential, $\overline{TB}$ and $\overline{TB}^*$ are the cumulated home and foreign trade balances. Note that Hooper and Morton (1982) allow for heterogeneous influences of the domestic and foreign cumulated trade balances in determining the exchange rate. I follow Meese and Rogoff's (1983) specification assuming the domestic and foreign variables affect the exchange rate with coefficients of equal magnitude but opposite sign. Eq. (9) is a quasi-reduced form, in a sense that it contains only contemporaneous explanatory variables on the right-hand side instead of expected future fundamentals. The exchange rate, nevertheless, does depend on market expectations about future fundamentals since these expectations are embodied in the interest differential and the expected inflation differential, as noted by Meese and Rogoff (1988).

Although the Hooper-Morton model nests a series of other monetary approach models, such as the flexible-price (Frenkel-Bilson) model and the sticky-price (Dornbusch-Frankel) or “overshooting” model, they are economically different in a theoretical perspective. Unlike the monetary approach which takes the exchange rate as the relative price two moneys, the portfolio approach views the exchange rate as the relative price of bonds (assets). The monetary approach assumes perfect substitutability between domestic and foreign securities, as a consequence asset holders are indifferent as to which they hold, and in turn this implies uncovered interest parity (UIP). In contrast, domestic and foreign assets are imperfect substitutes according to the portfolio-balance theorists, and thus holding different assets induces a risk premium which intrudes on the uncovered interest parity condition. According to the Eq. (9), the exchange rate is determined by the supply and demand for all foreign and domestic assets, not just the supply and demand for money as in the monetary approach. A surplus in the trade balance raises the supply of foreign assets and thus reduces their prices, which is an appreciation of the domestic currency.

Similar with the RID model, the Hooper-Morton model receives little empirical buttress. Meese and Rogoff (1983), for example, show that the portfolio balance model along with a range of other monetary models fails to outperform a simple random walk in forecasting exchange rate at horizons within one year. Subsequent studies by Alexander and Thomas (1987) and Gandolfo, Padoan, and Paladino (1990) further confirm and update the results of Meese and Rogoff. One exception is the finding of Somanth (1986), who suggests that introducing the lagged dependent variable among the explanatory variables improves the forecastability of the model, which indicates a delayed adjustment of the spot exchange rate to its equilibrium value as given by Eq.

(9). Regardless of these controversies, it shall be of interest to examine how these macroeconomic determinants affect the dynamics of the exchange rate through the transition probabilities as the present study does.

3. Data, Estimation, and Forecast

3.1. Data Description

The data set used in this study comprises quarterly observations for four bilateral nominal exchange rates: the Australian dollar (AUD), the Canadian dollar (CAD), the British pound (GBP), and the Japanese yen (JPY). Accordingly, five sets of macroeconomic measurements from these four countries plus the U.S. are employed: money supply, real gross domestic product, consumer price index, short-term and long-term interest rates, and trade balance (or current account balance). The data are mainly drawn from the IMF's International Financial Statistics (IFS). All exchange rates are U.S. dollar (USD) priced, i.e. the amount of USDs per unit of foreign currency. To be comparable in terms of unit measurement, the JPY is scaled by multiplying by 100.

The sample contains 138 end-of-quarter observations over the post Bretton-Woods period from the first quarter of 1973 to the second quarter of 2007. As regards the money supply variable, I use M1 for Japan and the U.S., M3 for Australia and Canada, and M4 for the U.K.. Australia M1 is not available for early years prior to 1975 while Canadian M1 is not available for the most recent years in the IFS database. The money aggregate measurements for the U.K. in the IFS database are M0 and M4, with M0 discontinued April 2006 and with M4 unavailable for early periods before the third quarter of 1982. As a result, the British M4 is drawn from the Statistical Interactive Database in the Bank of England. The real GDP is obtained through deflating the nominal GDP by GDP deflator with base year of 2000 (=100). Short-term and long-term interest rates are measured by 3-month treasury bill rate and long-term government bond yield rate (10 years or beyond). The Japanese and Australia treasury bill rates are taken from the Global Financial Data. Since no trade balance or current account balance is presented before 1977, the fundamental value for the Japanese yen based on the portfolio balanced model is estimated through 1977:Q1 to 2007:Q2.

The aggregate variables, money supply and real GDP, are seasonally adjusted while the rest of variables are seasonally unadjusted. Money supply and real GDP are measured by local currency while the trade balance is measured by the U.S. dollar. In estimation, money supply, income, and price level will be taken in the form of logarithm while the trade balance usually containing negative values is not able to be logarithmized and thus simply demeaned.

3.2. Estimation of the Time-Varying Markov-Switching ARCH

Let $Y_t = (y_t, \dots, y_{t-1}, y_1)'$ and $Z_{t-1} = (z'_{t-1}, z'_{t-2}, \dots, z'_0)'$ be vectors containing observations observed through date t, $S_t = (s_t, s_{t-1}, \dots, s_1)'$ historical realizations of state variables up to time t, and $\theta = (\mu', \alpha', \beta', a', b', \rho)'$ be the vector of model parameters, where $\mu = (\mu_1, \mu_2)'$ is the mean of change in the exchange rate, $\alpha = (\alpha_1, \alpha_2)'$ and $\beta = (\beta_1, \beta_2)'$, are intercepts and coefficients from ARCH, $a = (a_0, a_1, a_2)'$ and $b = (b_0, b_1, b_2)'$ are parameters in the transition probabilities, ρ is the unconditional probability of being in state 1 at the initial period, or $\rho = P(S_0 = 1; a, b)$.

Given specification described in equations (1), (3), and (4), the sample likelihood function is constructed as

$$L(\theta; Y_T) = \prod_{t=1}^{T} f(y_t | Z_{t-1}; \theta) \quad (10)$$

where

$$f(y_t|Z_{t-1};\theta) = \sum_i^2 \sum_j^2 f(y_t, s_t = j, s_{t-1} = i|Z_{t-1};\theta)$$

$$= \sum_i^2 \sum_j^2 f(y_t, s_t = j, s_{t-1} = i, Z_{t-1};\theta) \cdot P(s_t = j, s_{t-1} = i|Z_{t-1};\theta) \quad (11)$$

The weighting probability in (11) is computed recursively by applying Bayes' Rule given the initial unconditional probabilities ρ.[7]

$$P(s_t = j, s_{t-1} = i|Z_{t-1};\theta)$$

$$= P(s_t = j, s_{t-1} = i|Z_{t-1};\theta) \cdot P(s_{t-1} = i|Z_{t-1};\theta)$$

$$= p_t^{ij} P(s_{t-1} = i|Z_{t-1};\theta) \quad (12)$$

$$P(s_t = j|Z_t;\theta)$$

$$= P(s_t = j|Z_{t-1};\theta)$$

$$= \frac{\sum_i f(y_t|s_t=j,s_{t-1}=i,Z_{t-1};\theta) \cdot P(s_t=j,s_{t-1}=i|Z_{t-1};\theta)}{\sum_i \sum_j f(y_t|s_t=j,s_{t-1}=i|Z_{t-1};\theta) \cdot P(s_t=j,s_{t-1}=i|Z_{t-1};\theta)}$$

$$= \frac{\sum_i f(y_t|s_t=j,s_{t-1}=i,Z_{t-1};\theta) \cdot P(s_t=j,s_{t-1}=i|Z_{t-1};\theta)}{f(y_t|Z_{t-1};\theta)} \quad (13)$$

And the density of y_t conditional on s_t and s_{t-1} is:

$$f(y_t|s_t = j, s_{t-1} = i, Z_{t-1};\theta) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(\frac{-(y_t-\mu_j)^2}{2\sigma_t^2}\right) \quad (14)$$

where

$$\begin{cases} \sigma_t^2 = \alpha_1 + \beta_1(y_{t-1} - \mu_1)^2 \; for \; s_t = 1, s_{t-1} = 1 \\ \sigma_t^2 = \alpha_2 + \beta_2(y_{t-1} - \mu_1)^2 \; for \; s_t = 2, s_{t-1} = 1 \\ \sigma_t^2 = \alpha_1 + \beta_1(y_{t-1} - \mu_2)^2 \; for \; s_t = 1, s_{t-1} = 2 \\ \sigma_t^2 = \alpha_2 + \beta_2(y_{t-1} - \mu_2)^2 \; for \; s_t = 2, s_{t-1} = 2 \end{cases} \quad (15)$$

[7] Diebold, Lee, and Weinbach (1994) point out that ρ is determined by the parameters in the transition probabilities and thus not an additional parameter in the stationary case while it needs to be estimated separately in the nonstationary case. The stationarity is to be checked in the empirical analysis in the subsequent section.

In practice, construction and numerical maximization of the sample log-likelihood function involves summing over all possible values of $(s_{1,}s_2, \dots, s_T)$, which is computationally intractable, as $\left(s_{1,}s_2, \dots, s_T\right)$ may be realized in k^T ways. To this end, a version of the Expectation-Maximization (EM) algorithm proposed by Hamilton (1990) is typically employed to obtain the maximum likelihood estimation.

Given the smoothed state probabilities, $P(s_t = j, s_{t-1} = i|Y_T, Z_{t-1}; \theta), t = 2,3, \dots, T$, which are the inferred probabilities based on the entire sample. The maximum likelihood estimation for parameters can be obtained through differentiating the likelihood function. The first-order conditions for μ, α, and β are given:

$$\mu_k : \sum_{t=2}^{T} \left\{ \hat{\xi}_{t|T}^{ij} \cdot \sum_i^2 \sum_i^2 \left(\frac{\partial H_t^{ij}}{\partial \mu_k} + \frac{\partial H_L^{ij}}{\partial \mu_k} \right) \right\} = 0 \tag{16}$$

$$\alpha_k : \sum_{t=2}^{T} \left\{ \hat{\xi}_{t|T}^{ij} \cdot \sum_i^2 \sum_i^2 \left(\frac{\partial H_t^{ij}}{\partial \alpha_k} + \frac{\partial L_t^{ij}}{\partial \alpha_k} \right) \right\} = 0 \tag{17}$$

$$\beta_k : \sum_{t=2}^{T} \left\{ \hat{\xi}_{t|T}^{ij} \cdot \sum_i^2 \sum_i^2 \left(\frac{\partial H_t^{ij}}{\partial \beta_k} + \frac{\partial L_t^{ij}}{\partial \beta_k} \right) \right\} = 0 \tag{18}$$

where $k = 1, 2$, and

$$\hat{\xi}_{t|T}^{ij} = P(s_t = j, s_{t-1} = i|Z_{t-1}; \theta)$$

$$H_t^{ij} = -\log(\alpha_i + \beta_i(y_{t-1} - \mu_j)^2 \ )$$

$$L_t^{ij} = -\frac{(y_{t-1} - \mu_i)^2}{\alpha_i + \beta_i(y_{t-1} - \mu_j)^2}$$

for $i = 1, 2$. The first-order conditions for a and b are given:

$$a : \sum_{t=2}^{T} z_{t-1} \left\{ \hat{\xi}_{t|T}^{11} - p_t^{11} \cdot \zeta_{t-1|T}^{1} \right\} = 0 \tag{19}$$

$$b : \sum_{t=2}^{T} z_{t-1} \left\{ \hat{\xi}_{t|T}^{22} - p_t^{22} \cdot \zeta_{t-1|T}^{2} \right\} = 0 \tag{20}$$

where

$$\zeta_{t-1|T}^{i} = P(s_{t-1} = i|Y_{T,}Z_{T-1}; \theta)$$

for $i = 1, 2$. And the first-order condition for the initial unconditional probability, ρ, is given

$$\rho = P(s_t = 1 | Y_T, Z_{T-1}; \theta) \tag{21}$$

Note that the first-order conditions for the coefficients except ρ are nonlinear. Following Diebold, Lee, and Weinbach (1994), the close-form solutions are found by linearly approximation.

3.3. Auxiliary Estimation for Macroeconomic Models

Most macroeconomic aggregates and financial time series are nonstationary. It is well known that OLS regression among nonstationary time series is quite likely to produce spurious results (see Granger and Newbold, 1974; Phillips, 1986). One routine method to cure spurious regression is to difference the data before estimating the relation.

To obtain the fundamental value of the exchange rate in Eq. (8) and Eq. (9), I estimate the following regression based on a first-difference specification:

$$\Delta e_t = \Delta X_t \cdot \Pi + u_t \tag{22}$$

where Δe_t is the change in the log exchange rate, ΔX_t is the first-difference of the vector of relative fundamental variables under consideration. The fundamental value is thus constructed based on the estimated parameters, $\widehat{\Pi}$.

3.4. Forecast

According to the two-state Markov-switching ARCH model described in (1), and given the maximum likelihood estimates, $\hat{\theta}$, it is straightforward to compute the h-period-ahead forecast of y_{t+h}, on the basis of observation of y through time t,

$$\widehat{y_{t+h}} = E\left[y_{t+h} | Y_t, Z_{t-1}; \hat{\theta}\right] = E\left[\mu_{s_{t+h}} + u_{t+h} | Y_t, Z_{t-1}; \hat{\theta}\right] = \hat{\mu}' \cdot \hat{\zeta}_{t+h|t} \tag{23}$$

where

$$\hat{\mu} = \begin{pmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \end{pmatrix},\ \zeta_{t+h|t} = \prod_{j=1}^{h} P_{t+j|t} \cdot \zeta_{t|t} \tag{24}$$

and

$$\zeta_{t|t} = \begin{pmatrix} P(s_t = 1|Y_T, Z_{T-1}; \hat{\theta}) \\ P(s_t = 2|Y_T, Z_{T-1}; \hat{\theta}) \end{pmatrix} \tag{25}$$

and $P_{t+j|t}$ is the projected transition probability matrix at time $t+j$, $j = 1,2, \dots, h$, which is given

$$P_{t+j|t} = \begin{pmatrix} P_{t+j|t}^{11} & P_{t+j|t}^{12} \\ P_{t+j|t}^{21} & P_{t+j|t}^{22} \end{pmatrix}$$

$$= \begin{pmatrix} P(s_{t+j} = 1|s_t = 1, Z_{T-1}; \hat{\theta}) & P(s_{t+j} = 2|s_t = 1, Z_{T-1}; \hat{\theta}) \\ P(s_{t+j} = 1|s_t = 2, Z_{T-1}; \hat{\theta}) & P(s_{t+j} = 2|s_t = 2, Z_{T-1}; \hat{\theta}) \end{pmatrix} \tag{26}$$

The h-period-ahead forecast of u_{t+h}^2, given the observed values, Y_t and Z_{t-1}, the hypothetical history of S_t, and the maximum likelihood estimate $\hat{\theta}$, can be calculated as

$$\begin{aligned} \hat{\sigma}_{t+h}^2 &= E\left[u_{t+h}^2 | Y_t, Z_{t-1}, S_t; \hat{\theta}\right] \\ &= E\left[\sigma_{t+h}^2 v_{t+h}^2 | Y_t, Z_{t-1}, S_t; \hat{\theta}\right] \\ &= E\left[\sigma_{t+h}^2 | Y_t, Z_{t-1}, S_t; \hat{\theta}\right] \\ &= E[\alpha_{s_{t+h}} + \beta_{s_{t+h}}\left(y_{t+h-1} - \mu_{S_{t+h-1}}\right)^2 | Y_t, Z_{t-1}, S_t; \hat{\theta}] \\ &= \hat{\Sigma}' \cdot \hat{\xi}_{t+h|t} \end{aligned} \tag{27}$$

where

$$\hat{\Sigma} = \begin{pmatrix} \hat{\alpha}_1 + \hat{\beta}_1\left(\hat{y}_{t+h-1|t} - \mu_1\right)^2 \\ \hat{\alpha}_2 + \hat{\beta}_2\left(\hat{y}_{t+h-1|t} - \mu_1\right)^2 \\ \hat{\alpha}_1 + \hat{\beta}_1\left(\hat{y}_{t+h-1|t} - \mu_2\right)^2 \\ \hat{\alpha}_2 + \hat{\beta}_2\left(\hat{y}_{t+h-1|t} - \mu_2\right)^2 \end{pmatrix}$$

and

$$\hat{\xi}_{t+h|t} = \begin{pmatrix} P(s_{t+h} = 1, s_{t+h-1} = 1|Y_t, Z_{T-1}; \hat{\theta}) \\ P(s_{t+h} = 2, s_{t+h-1} = 1|Y_t, Z_{T-1}; \hat{\theta}) \\ P(s_{t+h} = 1, s_{t+h-1} = 2|Y_t, Z_{T-1}; \hat{\theta}) \\ P(s_{t+h} = 2, s_{t+h-1} = 2|Y_t, Z_{T-1}; \hat{\theta}) \end{pmatrix} = \begin{pmatrix} P^{11}_{t+j|t} \cdot \hat{\zeta}^{1}_{t+h-1|t} \\ P^{12}_{t+j|t} \cdot \hat{\zeta}^{1}_{t+h-1|t} \\ P^{21}_{t+j|t} \cdot \hat{\zeta}^{2}_{t+h-1|t} \\ P^{22}_{t+j|t} \cdot \hat{\zeta}^{2}_{t+h-1|t} \end{pmatrix} \tag{28}$$

It is noteworthy that, calculating the projected transition probability matrix $P_{t+j|t}$ calls for to predict the future values of the exogenous variable Z_{t+h-1} for $h > 1$, which essentially requires further to set up a forecasting framework for Z_t. Additional modeling for the exogenous variables will necessarily induce extra uncertainty in forecasting the future change in exchange rate and future volatility. For this end, I treat the P_t as constant over the periods of time t through $t + h$.

4. Empirical Results

4.1. Preliminary Analysis of the Data

The exchange rate determination models depict the equilibrium relationship between the exchange rate and its macroeconomic determinants. It is of particular interest to investigate whether certain linkage exists empirically in the actual data.

Table 1 presents unit root tests for four currencies and their macroeconomic determinants. Both the augmented Dickey-Fuller and Phillips-Perron procedures are employed to check the stationarity of these time series, with the null hypothesis that each of the series contains a unit root. The overall test results are consistent with what have reported in the literature - nominal exchange rates, relative money supplies, relative incomes, relative price levels, interest rate differentials, and trade balance differentials are generally nonstationary. In fact, most null hypotheses of unit root cannot be rejected at the 5% or 10% significance level, as indicated by high p-values. The nonstationarity of the short-term interest rate differentials is relatively less evident. The p-value for the differentials between US and UK, for example, is 0.001 according to the augmented Dickey-Fuller test and 0.018 according to the Phillips-Perron test, both indicating a strong rejection of the null hypothesis at 5% significance level. The relative price level between US and UK is special in a sense that it exhibits significant stationarity while the relative price levels for other countries are significantly nonstationary. As regards the first difference of the exchange rate, no unit root is found for all four currencies, indicating that the levels of log exchange rates are very likely to be $I(1)$.

[Insert Table 1 here]

Cointegration, the notion that a linear combination of two or more nonstationary series may be stationary, as pointed out by Engle and Granger (1987), is of particular importance for the existence of a stable, linear relationship between the exchange rate and the relevant fundamental variables. Table 2 reports results based on the multivariate cointegration test suggested by Johansen (1988, 1991) and Johansen and Juselius (1990, 1992).[8] These results suggest that there is at least one cointegrating equation at 5% significance level for all four currencies. Particularly, there may be four cointegrating relationships for the Japanese yen and its relevant macroeconomic determinants,[9] while there is only one cointegrating equation statistically verified for the British pound. Table 2 also presents one cointegrating equation for

[8] There are two different test statistics called the Trace and λmax. Critical values and p-values are taken from MacKinnon, Haug, and Michelis (1999) who calculate these values by using response surfaces and claim that these values are extremely accurate relative to previously used ones suggested by Osterwald and Lenum (1992) and Johansen (1995).

[9] Although the λmax test shows that there is only one cointegrating equation, but the p-value in testing $r \leq 1$ is 0.051, which is negligibly above the 5% significance level, and the p-values in testing $r \leq 2$ and $r \leq 3$ are 0.028 and 0.043, respectively. Relatively, the λmax test is more likely to reject the null hypothesis of cointegrating relationship among exchange rates and their fundamental.

each currency (with the coefficient for the exchange rate normalized to unity). Most of the coefficients for these equations are statistically significant, implying that there exists long-run relationship between the exchange rate and the fundamental variables. This finding is consistent with some extant literature that shows that the equilibrium level of the exchange rate may pin down by the macroeconomic aggregates in the long-run.

[Insert Table 2 here]

Since its introduction by Engle (1982), the ARCH-family models have been widely used in various branches of econometrics, especially in financial time series analysis.[10] Table 3 presents a simple ARCH test based on Engle's LM test. As we can see, the ARCH effects in these exchange rates are unclear. Among the four currencies examined, the Australian dollar shows apparent autoregressive conditional heteroskedasticity when two or three periods' lagged variances are considered in the specification of the conditional volatility. In the meanwhile, no significant evidence of ARCH effects in other currencies. However, one has to be cautious in interpreting these results since they may not imply that the exchange rate does not contain the feature of ARCH. Hamilton and Susmel (1994) point out the financial time series like exchange rate may be related to regime changes in the ARCH process. The present simple ARCH test imposes a single-state on the ARCH process, and thus the results can be misleading to a great extent. In addition, the quarterly data set involves relatively insufficient number of observations, which may seriously destroy the temporal dependence in the second-order moments of the time

[10] See Bollerslev, Chou, and Kroner (1992), Bollerslev, Engle, and Nelson (1994), Shephard (1996), Bauwens, Laurent, and Rombout (2006), among others.

series. In fact, it is generally recognized that estimating ARCH-type models requires large samples but the high frequency data of macroeconomic variables are essentially unavailable.

[Insert Table 3 here]

4.2. Estimates of the Fundamentals-based models

Table 4 shows the estimates of the real interest differential (RID) model and the portfolio balance (Hooper-Morton) model. Following Cheung, Chinn, and Pascual (2005), I estimate these two models based on a first-difference specification which as the authors point out emphasizes the effects of changes in the macro variables on exchange rates. As expected, the coefficients of the macro variables are generally insignificant with the exception of the relative money supply on the Japanese yen. This is consistent with the conventional wisdom that the contemporaneous fundamentals are of little explanatory power in describing the variation of the spot exchange rates. Nevertheless, as discussed in section 2, the fundamentals-based models essentially specify the long-run equilibrium level the nominal exchange rates and thus the estimates in effect determine the fundamental values for the nominal exchange rates. Figure 1 depicts the dynamics of the spot rates (to save space, only the British pound is reported), their fundamental values predicted by various macro models, and the deviations of the exchange rates from the fundamental values. As one can see, the fitted values (fundamental values) display poor goodness-of-fit but they do specify long-run trends for these spot rates, notwithstanding large and extremely volatile deviations in the short run.

[Insert Table 4 here]

[Insert Figure 1 here]

Table 5 further investigates the stationarity of the deviations of the spot rates from their fundamental values. In the present analysis, these deviations are "news" for market participants to form the beliefs on the transition probabilities. According to Diebold, Lee, and Weinbach (1994), it is important to differentiate the cases of stationarity and nonstationarity of the variables affecting the evolution of the transition probabilities. Particularly, in the nonstationary case, the unconditional probability of being state 1 (or 2) at the initial period would be an additional parameter to be estimated. Like the unit root tests for the exchange rates and macro variables, the augmented Dickey-Fuller and Phillips-Perron testing procedures are employed. The results show that these deviations are generally nonstationary as the null hypotheses of containing a unit root are strongly rejected in most cases. Two exceptions emerge in the British pound. The p-values are lower than the 5% significance level for the deviations from the RID model and the Hooper-Morton model. This may also be verified in Figure1 where RID model and the Hooper-Morton model present a relatively better goodness-of-fit for the British pound. In estimation, without loss of generality, I let the data endogenously determine the unconditional probability of being state 1 at the initial period, i.e. all cases are viewed as nonstationary.

[Insert Table 5 here]

4.3. MLE of the Time-Varying Markov-Switching ARCH

Table 6 reports maximum likelihood estimates of the two-state time-varying Markov-switching ARCH model. Across all specifications, the estimated mean changes in the exchange rates are generally statistically significant. Particularly, the average quarterly depreciation rate in the downward movement regime (state 1) is -1.99 percent for the Australian dollar, -0.86 percent for the Canadian dollar, -2.05 percent for the Japanese yen, and -4.83 percent for the British pound while the average quarterly appreciation rate in the upward movement regime (state 2) is 1.08 percent for the Australian dollar, 1.02 percent for the Canadian dollar, 3.30 percent for the Japanese yen, and 1.69 percent, respectively. As regards the estimated mean changes in the exchange rates, four specifications produce relatively stable results for the Canadian dollar and the British pound. The results, nevertheless, vary tremendously when different macro models are considered for the Australian dollar and the Japanese yen. The depreciating rate, for example, is ranging from -0.7 percent to -3.0 percent per quarter for the Australian dollar and the appreciation rate is ranging from 1.8 percent to 6.1 percent for the Japanese yen.

[Insert Table 6 here]

The coefficients on the ARCH term are of more interest. The point estimates show that the exchange rates are very likely to contain ARCH effects in the variance structure as the coefficients are mostly significant. For example, the estimates of the coefficients on the ARCH term based on the portfolio balance model (Hooper-Morton model) statistically differ against zero for all currencies across both states. The ARCH effects seem to be more evident in the appreciation state for the Canadian dollar, the Japanese yen, and the British pound as all coefficient estimates are significant in state 2 while opposite results are found for the Australian dollar. The positive evidence is consistent with the earlier finding of Diebold (1988) who has documented strong ARCH effects in all seven nominal dollar spot exchange rates. Combining the ARCH tests presented the previous subsection, this finding also further supports the argument by Lamoureux and Lastrapes (1990) and Perron (1989) that the ARCH process may subject to regime change. The regime-switching ARCH effects can also be seen in Figure2. Generally, the alternation of the low-variance and high-variance regimes is clearly distinguished for these currencies. For example, the Canadian dollar and the Japanese yen seem to be more volatile since the late 90's while the British pound has strikingly high variance during the mid of 80's and the mid of 90's. In addition, the low-variance regime tends to be more prolonged with relatively more stable variance in terms of magnitude for all currencies.

[Insert Figure 2 here]

The rest of estimates measure the effect of exogenous variables including the observed deviations of the exchange rate from its fundamental value determined by relevant macroeconomic determinants. Although the results are fairly mixing, the fundamentals substantially affect the transition probabilities in many cases. Under the purchasing power parity model, both coefficients are significant in the logistic function of the transition probabilities for the Australian dollar. Similarly, the deviation from the fundamental value specified by Mark (1995) has strong transitional effects on the Canadian dollar.

The transitional effects of macroeconomic determinants are further manifested by the staying probabilities, $\Pr(s_t = i | s_{t-1} = i, Z_{t-1}, \theta)$, and the smoothed probabilities, $\Pr(s_t = i | Y_t, Z_{T-1}, \theta)$, as plotted in Figure 3 and Figure 4, respectively (only the British pound is reported here). The staying probability, namely, refers to the probability that once the data process enters certain state, it will stay in that state for the next period. Intuitively, when the data process is stable within some regime, that is, it stays in that state for a prolonged period, the staying probability would be close to one. On the other hand, if the time series is very volatile, the staying probability would be close to zero, which means the data process shifts between different states very frequently. As one can see, Figure 3 roughly shows this pattern. For example, the staying probabilities in the Japanese yen and British pound are above 0.5 for most of times across all specifications, which is consistent with the Engel and Hamilton's (1990) finding that there are long swings in exchange rate process.[11] Nevertheless, it is important to note that the transition probabilities are sensitive to the observed exogenous variables. If the observed deviations or previous change in the spot rate vary a lot, the possibility for staying in the same state next period would be low, which in turn implies that staying probabilities are very low while the shifting probabilities are close to one. Figure 4 plots the inferred probabilities of the unobserved state variables based on the entire sample. Introducing the time-varying effects of the macroeconomic determinants makes the smoothed probabilities sensitive to variations in exchange rates.

[Insert Figure 3 here]

[Insert Figure 4 here]

[11] See also Dewachter (1997), Klaassen (1999), and Cheung and Erlandsson (2005).

4.4. Diagnostic Analysis of Specification

One of the most natural and important tests associated with Markov-switching models is to test whether the data best characterized by a single state or two (or multiple) states. Under the null hypothesis of only one state, however, the transition probabilities of the Markov-switching process are unidentified, which makes the standard regularity conditions for the asymptotic tests of the null hypothesis no longer valid. Many researchers have proposed various alternative testing procedures to tackle this issue and documented that exchange rates tend to follow multi-regime process.[12] The main focus of this paper is not to establish the existence of multiple regimes in the dynamics of exchange rates, but rather to understand whether there are ARCH effects in the error process of the exchange rates, whether the conditional variance is subject to regime shifts, and whether the macroeconomic determinants possibly have transitional effects on the evolution of data process. To this end, the present analysis assumes that the mean change in the exchange rates follows two states, which thus sidesteps the methodological issue of unidentification and in turn justifies the asymptotic tests.

The first diagnostic test is against the null hypothesis of no ARCH effects in the exchange rates which restricts the coefficients on the state-dependent ARCH term, β_1, β_2, to be zero. Under the null hypothesis, the model reduces to the framework described by Diebold, Lee, and Weinbach (1994), in which mean and variance are state-dependent but constant over time within each regime. The first two columns of Table 7 present the likelihood ratio statistics and relevant asymptotic χ^2 p-values for this test. As we can see, the null hypotheses for various specifications are rejected in most cases at 5% significance level with exceptions including Hooper-Morton model for the Australian dollar and the Canadian dollar, and both PPP model

[12] See Engel and Hamilton (1990), Engel (1994), and Klaassen (1999).

and Mark's specification for the British pound. According to the RID model, the null hypothesis of no ARCH effects is easily rejected for all currencies. The ARCH effects in the Japanese yen's error structure seem to be fairly strong, irrespective of whichever the macroeconomic model is considered.

[Insert Table 7 here]

The next test analyzes the question whether there is regime change in the ARCH process. The null hypothesis imposes a single state on the intercepts and coefficients of the conditional variance: $\alpha_1 = \alpha_2, \beta_1 = \beta_2$. Note that this null hypothesis admits that there may be ARCH effects in the conditional variance but distinguishes neither high-variance nor low-variance. The results of the Table 7 show that the regime shifts in the ARCH process are strongly favored in the cases of the Canadian dollar and the Japanese yen while slightly weaker evidence is found for the rest of currencies. The null hypothesis, for instance, is easily rejected in the Japanese yen across all specification while in the case of the British pound the ARCH effects are statistically justified under the RID model and Mark's specification but are not well established in other specifications.

The third diagnostic test considers whether the exogenous variables including macroeconomic determinants have transitional effects on the evolution of exchange rates. Under the null hypothesis, the model reduces to a Markov-switching ARCH framework with constant transition probabilities. Thus the restricted model is common for all the four specifications associated with macro models. The last two columns of Table 7 present the empirical results for this test. The low p-values of the empirics clearly favor a time-varying version Markov-switching ARCH framework as all specifications across all currencies reject the null at 5% significance level, with only two exceptions--Mark's specification in the Australian dollar and portfolio balance model in the Japanese yen. Roughly speaking, this concludes that fundamental variables, like money supply, income, interest rate differentials, and trade balances, can potentially affects the dynamics of exchange rates in a nonlinear way, say through the transition probabilities of a Markovian process as described in the present analysis.

4.5. Forecast Performance

It has been a convention to examine the forecast performance of any empirical model of exchange rates relative to a simple random walk specification since Meese and Rogoff's (1983) seminal study. Consensus has admitted that achieving superior forecast accuracy to the random walk is extremely difficult, especially at short horizons.[13] One notable exception is the study by Engel and Hamilton (1990), who propose a two-state Markov-switching model to capture the long swings of the quarterly exchange rates and show that their model generates better forecasts than a random walk over short horizons. In fact, their study has popularized modeling exchange rates using Markov-switching framework.

Table 8 presents the short-horizon forecast performance of the time-varying Markov-switching ARCH model. Following the convention, I measure the forecast accuracy in terms of mean squared errors (MSE). Table 8 reports the MSE ratio which is the ratio of the MSE from a competing model relative to that of a simple random walk benchmark. A value of MSE ratio lower than one means the relevant model outperforms the random walk. P-values are reported as well based on Diebold and Mariano (1995). The Diebold-Mariano (DM) statistic tests the

[13] Many researchers have documented the predictability of exchange rates over the long horizons. See, for example, MacDonald and Taylor (1994), Chinn and Meese (1995), Mark (1995), Mark and Choi (1997), Groen (2000), and Mark and Sul (2001). In the meanwhile, other economists like Kilian (1999), Berkowitz and Giorianni (2001), and Rapach and Wohar (2002) argue that exchange rates not predictable with monetary models.

significance of the difference between the forecast MSEs of the competing model and the random walk. As regards the in-sample forecasts, the MSE ratios are clearly favorable to the time-varying Markov-switching ARCH model. All specifications are almost uniformly outperforming the random walk across all currencies, with a great part of the DM p-values lower than 5% significance level. The out-of-sample forecast results, however, are ambiguous. In effect, no consistent patterns are revealed in terms of outperformance. The PPP specification, for example, achieves better forecasts in the cases of the Australian dollar and the Canadian dollar, but fails to beat the random walk for the Japanese yen and the British pound. The portfolio balance model delivers superior forecast accuracy both at one- and two-quarter ahead forecast in the case of the Japanese yen while only outperform the random walk at one-quarter ahead forecast for the rest of currencies.

[Insert Table 8 here]

It is important to note that the superior evidence of in-sample forecastability of the time-varying Markov-switching ARCH model should not be discounted given the less convincing out-of-sample forecast performance. Conventional wisdom suggests that out-of-sample results are more reliable than in-sample results as the latter tends to suffer from data mining and is biased in favor of detecting spurious forecastability. This notion, however, is seriously challenged by Inoue and Kilian (2002). Inoue and Kilian show that in-sample and out-of-sample tests of forecastability are asymptotically equally reliable under the null of no forecastability in an environment free from data mining. On the other hand, in-sample tests tend to reject the null hypothesis of no forecastability more often than out-of-sample test in practice. As a consequence, they conclude that results of in-sample tests of forecastability will typically be more credible than results of out-of-sample tests.

5. Conclusion

This paper considers a nonlinear exchange rate model in the context of Markov-switching by allowing for macroeconomic fundamental variables affecting the transition probabilities. Four macroeconomic models which theoretically specify the fundamental value of the nominal exchange rate are examined: the purchasing power parity, Mark's (1995) specification, the real interest differential (RID) model, and the portfolio balance model (Hooper-Morton model). The maximum likelihood estimates and diagnostic analyses suggest that the macroeconomic determinants can largely affect the dynamics of exchange rates nonlinearly through the transition probabilities in a Markovian process.

My analysis further examines the effects of the autoregressive conditional heteroskedasticity (ARCH) in the error processes of exchange rates. The ARCH effects are not well identified in the preliminary analysis which imposes a single state of the data process but in an environment distinguishing regimes of low-variance and high-variance, these effects are fairly strong across all major dollar-priced exchange rates. This positive evidence indicated by the time-varying Markov-switching ARCH model is consistent with previous finding that financial time series, such as stock returns and exchange rates, tend to follow ARCH process but are subject to regime change.

Both in-sample and out-of-sample forecast performance are investigated as a conventional test for empirical modeling. Relative to the random walk benchmark, superior in-sample forecast accuracy of the proposed framework is well documented while mixing results are found in terms of out-of-sample forecastability. It is important to note that although out-of-sample forecastability is more favored by the convention wisdom, the quality of in-sample forecastability may be more valuable in practice according to recent finding by Inoue and Killian (2002).

In the perspective of modeling, no specification based on four prevailing macroeconomic models is superior to one another. This buttress the notion that the exact nature of the exchange rate dynamics is quite complex and macro fundamental variables may only account for part of the behavior of the spot rates. Other factors, like microstructure effects and unobservable trend components,[14] may also be important determinants of the exchange rate behavior.

[14] Lyons (2001), Sarno and Taylor (2001), and Evans and Lyons (2002) suggest that microstructure effects like order flow may account for the behavior of exchange rates.

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Table 1. Unit Root Test for Exchange Rates and Relative Macro Variables

Variables	Augmented Dickey-Fuller[a]		Phillips-Perron[b]	
	t-stat	p-value[c]	Adjust. t-stat	p-value[c]
Australian Dollar				
e	-2.042	0.269	-2.023	0.277
Δe	-7.509	0.000	-11.616	0.000
m-m*	-0.366	0.910	-0.606	0.864
q-q*	-1.774	0.392	-1.770	0.394
p-p*	-2.567	0.103	-2.949	0.042
i^s-i^{s*}	-2.395	0.145	-3.246	0.020
i^l-i^{l*}	-2.661	0.084	-2.960	0.041
TB-TB^*	2.464	1.000	1.858	1.000
Canadian Dollar				
e	-1.530	0.515	-1.735	0.411
Δe	-10.655	0.000	-10.993	0.000
m-m*	-1.403	0.579	0.000	0.979
q-q*	0.000	-0.950	-2.440	0.133
p-p*	-1.458	0.552	-1.541	0.510
i^s-i^{s*}	-2.700	0.077	-2.676	0.081
i^l-i^{l*}	-2.231	0.197	-2.285	0.178
TB-TB^*	2.830	1.000	1.886	1.000
Japanese Yen				
e	-1.323	0.618	-1.379	0.591
Δe	-10.548	0.000	-10.582	0.000
m-m*	0.449	0.984	0.122	0.966
q-q*	-0.627	0.860	-2.188	0.212
p-p*	-0.122	0.944	1.028	0.997
i^s-i^{s*}	-2.855	0.054	-2.613	0.093
i^l-i^{l*}	-2.458	0.128	-2.323	0.166
TB-TB^*	1.902	1.000	0.970	0.996
British Pound				
e	-2.496	0.119	-2.663	0.083
Δe	-10.038	0.000	-10.023	0.000
m-m*	-0.096	0.947	-0.731	0.834
q-q*	-2.246	0.191	-2.317	0.168
p-p*	-10.109	0.000	-4.723	0.000
i^s-i^{s*}	-4.154	0.001	-3.284	0.018
i^l-i^{l*}	-1.788	0.385	-1.790	0.384
TB-TB^*	1.165	0.998	0.915	0.996

Note: null hypothesis: time series has a unit root. [a] the number of lags is determined automatically based on SIC, [b] Newey-West using Bartlett kernel, [c]MacKinnon (1996) one-sided p-values. Constant mean is included in the test equation but no trend.

Table 2. Cointegration Test (Johansen maximum likelihood estimation)

Number of Cointegration Vectors	Trace			Maximum Eigenvalue (λ_{Max})		
	Statistic	Critical Value	p-value	Statistic	Critical Value	p-value
Australian Dollar						
$r = 0$	182.028	125.615	0.000	68.256	46.231	0.000
$r \leq 1$	113.772	95.754	0.002	45.460	40.078	0.011
$r \leq 2$	68.312	69.819	0.066	26.183	33.877	0.310
$r \leq 3$	42.129	47.856	0.155	17.859	27.584	0.507
$r \leq 4$	24.270	29.797	0.189	13.663	21.132	0.393
$r \leq 5$	10.607	15.495	0.237	10.207	14.265	0.199
$r \leq 6$	0.399	3.841	0.527	0.399	3.841	0.527

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level

$$e = \underset{(16.55)}{-70.60}(m-m^*) + \underset{(97.14)}{609.67}(q-q^*) \underset{(42.61)}{-106.25}(p-p^*) \underset{(6.76)}{-37,64}(i^s-i^{s^*}) \underset{(16.05)}{+108.39}(i^l-i^{l*)} + \underset{(0.07)}{0.31}(TB-TB^{*)}$$

Canadian Dollar						
$r = 0$	149.424	125.615	0.001	43.205	46.231	0.102
$r \leq 1$	106.219	95.754	0.008	38.181	40.078	0.081
$r \leq 2$	68.038	69.819	0.069	23.977	33.877	0.457
$r \leq 3$	44.061	47.856	0.109	21.768	27.584	0.233
$r \leq 4$	22.293	29.797	0.283	14.909	21.132	0.295
$r \leq 5$	7.384	15.495	0.533	7.383	14.265	0.445
$r \leq 6$	0.002	3.841	0.965	0.002	3.841	0.965

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level

Max-eigenvalue test indicates no cointegration at the 0.05 level

$$e = \underset{(15.34)}{-17.34}(m-m^*) - \underset{(128.30)}{831.11}(q-q^*) \underset{(75.52)}{-381.62}(p-p^*) + \underset{(8.20)}{26.24}(i^s-i^{s^*}) - \underset{(19.34)}{64.86}(i^l-i^{l*)} + \underset{(0.06)}{0.37}(TB-TB^{*)}$$

Note: r denotes the number of cointegrating relations (the cointegrating rank). The null hypothesis is no cointegration, p-values are taken from MacKinnon-Haug-Michelis (1999), and standard errors for coefficients in cointegration equation are in parentheses.

Table 2. Cointegration Test (cont'd)

Number of Cointegration Vectors	Trace			Maximum Eigenvalue (λ_{Max})		
	Statistic	Critical Value	p-value	Statistic	Critical Value	p-value
Japanese Yen						
$r = 0$	186.697	125.615	0.000	55.728	46.231	0.004
$r \leq 1$	130.969	95.754	0.000	39.984	40.078	0.051
$r \leq 2$	90.985	69.819	0.000	35.962	33.877	0.028
$r \leq 3$	55.023	47.856	0.009	28.126	27.584	0.043
$r \leq 4$	26.897	29.797	0.104	16.531	21.132	0.195
$r \leq 5$	10.367	15.495	0.254	9.492	14.265	0.248
$r \leq 6$	0.875	3.841	0.350	0.875	3.841	0.350

Trace test indicates 4 cointegrating eqn(s) at the 0.05 level

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

$$e = \underset{(31.50)}{83.35}(m-m^*) + \underset{(122.89)}{389.82}(q-q^*) \underset{(49.80)}{-363.38}(p-p^*) + \underset{(9.47)}{13.46}(i^s-i^{s^*}) \underset{(15.33)}{-23.54}(i^l-i^{l^*)}) - \underset{(022)}{0.54}(TB-TB^{*)})$$

British Pound						
$r = 0$	148.792	125.615	0.001	53.196	46.231	0.008
$r \leq 1$	95.595	95.754	0.051	36.787	40.078	0.112
$r \leq 2$	58.809	69.819	0.274	23.581	33.877	0.487
$r \leq 3$	35.227	47.856	0.436	16.109	27.584	0.657
$r \leq 4$	19.118	29.797	0.484	12.586	21.132	0.491
$r \leq 5$	6.532	15.495	0.633	5.889	14.265	0.628
$r \leq 6$	0.642	3.841	0.423	0.642	3.841	0.423

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level

$$e = \underset{(1102.43)}{-105.29}(m-m^*) + \underset{(840.26)}{1880.28}(q-q^*) + \underset{(261.48)}{1393.44}(p-p^*) - \underset{(40.00)}{81.30}(i^s-i^{s^*}) + \underset{(38.95)}{33.64}(i^l-i^{l^*)}) + \underset{(0.50)}{0.92}(TB-TB^{*)})$$

Note: *r* denotes the number of cointegrating relations (the cointegrating rank). The null hypothesis is no cointegration, p-values are taken from MacKinnon-Haug-Michelis (1999), and standard errors for coefficients in cointegration equation are in parentheses.

Table 3. ARCH Test of Exchange Rates

Exchange Rates	Random Walk: $e_t=\mu+e_{t-1}+\varepsilon_t$		AR(1): $e_t=\alpha+\beta e_{t-1}+\varepsilon_t$	
	Engle's LM test	p-value	Engle's LM test	p-value
Australian Dollar				
ARCH(1)	2.542	0.111	2.781	0.095
ARCH(2)	6.033	0.049	8.091	0.017
ARCH(3)	6.395	0.094	8.521	0.036
ARCH(4)	6.788	0.148	8.763	0.067
Canadian Dollar				
ARCH(1)	2.318	0.128	0.128	0.128
ARCH(2)	2.295	0.317	0.317	0.317
ARCH(3)	4.892	0.180	0.180	0.180
ARCH(4)	5.777	0.216	0.216	0.216
Japanese Yen				
ARCH(1)	0.896	0.344	1.194	0.275
ARCH(2)	1.315	0.518	1.367	0.505
ARCH(3)	3.193	0.363	3.492	0.322
ARCH(4)	3.079	0.545	3.345	0.502
British Pound				
ARCH(1)	0.268	0.605	0.059	0.808
ARCH(2)	2.387	0.303	2.105	0.349
ARCH(3)	2.750	0.432	2.253	0.522
ARCH(4)	3.331	0.504	3.443	0.487

Note: ARCH test is a Lagrange multiplier test based on Engle (1982). The null hypothesis is that there is no ARCH effect in the conditional variance.

Table 4. Estimates of Fundamentals-based Models

	RID model[a]			H-M model[b]		
	Coef.	Std	t-Stat	Coef.	Std	t-Stat
Australian Dollar						
con.	-0.670	0.494	-1.358	-0.604	0.501	-1.205
m-m*	-20.912	17.672	-1.183	-20.321	17.712	-1.147
q-q*	26.755	39.904	0.670	28.778	40.038	0.719
i^s-i^{s*}	1.878	1.297	1.448	1.934	1.300	1.487
i^l-i^{l*}	-2.814	3.069	-0.917	-2.810	3.073	-0.914
TB-TB^*	-	-	-	0.039	0.049	0.801
Canadian Dollar						
con.	-0.198	0.252	-0.786	-0.251	0.254	-0.990
m-m*	-15.262	11.836	-1.290	-15.115	11.795	-1.281
q-q*	-5.835	22.247	-0.262	-7.604	22.206	-0.342
i^s-i^{s*}	-0.179	1.412	-0.127	-0.270	1.409	-0.192
i^l-i^{l*}	0.159	3.165	0.050	0.052	3.155	0.016
TB-TB^*	-	-	-	-0.035	0.025	-1.388
Japanese Yen						
con.	0.822	0.499	1.647	0.746	0.556	1.343
m-m*	45.133	19.579	2.305	44.062	20.926	2.106
q-q*	9.049	11.386	0.795	-12.129	21.965	-0.552
i^s-i^{s*}	-1.824	2.863	-0.637	-2.168	3.141	-0.690
i^l-i^{l*}	-1.265	4.581	-0.276	-1.656	4.975	-0.333
TB-TB^*	-	-	-	-0.092	0.052	-1.774
British Pound						
con.	-0.393	0.547	-0.719	-0.512	0.549	-0.932
m-m*	-21.903	22.714	-0.964	-24.087	22.615	-1.065
q-q*	-43.645	44.201	-0.987	-45.465	43.945	-1.035
i^s-i^{s*}	-1.381	1.993	-0.693	-1.581	1.985	-0.797
i^l-i^{l*}	0.287	3.454	0.083	0.359	3.433	0.105
TB-TB^*	-	-	-	-0.077	0.048	-1.622

[a] RID model: real interest differential model, [b] H-M model: Hooper and Morton's portfolio balance model. Estimation is implemented based on first-difference specification to avoid spurious regression.

Table 5. Stationarity Test for Deviations of the Exchange Rate from its Fundamental value

	Augmented Dickey-Fuller[e]		**Phillips-Perron**[f]	
	t-stat	p-value[g]	Adjust. t-stat	p-value[g]
Australian Dollar				
PPP model[a]	-1.964	0.302	-2.369	0.153
Mark's specification[b]	-0.853	0.800	-1.212	0.668
RID model[c]	-2.379	0.150	-2.438	0.133
H-M model[d]	-2.563	0.103	-2.765	0.066
Canadian Dollar				
PPP model[a]	-1.448	0.557	-1.709	0.425
Mark's specification[b]	-0.077	0.949	-0.932	0.776
RID model[c]	-0.541	0.878	-0.996	0.754
H-M model[d]	-1.083	0.722	-1.576	0.492
Japanese Yen				
PPP model[a]	-1.762	0.398	-2.115	0.239
Mark's specification[b]	-1.444	0.559	-1.531	0.515
RID model[c]	-2.358	0.156	-2.333	0.163
H-M model[d]	-2.271	0.183	-2.664	0.083
British Pound				
PPP model[a]	-2.122	0.237	-2.481	0.122
Mark's specification[b]	-1.964	0.303	-2.333	0.163
RID model[c]	-3.137	0.026	-3.421	0.012
H-M model[d]	-3.044	0.033	-3.354	0.014

Deviations of the exchange rate and its fundamental value are defined as: $d_t = e_t - f_t$

[a] PPP Model: $f_t = p_t - p_t^*$

[b] Mark's (1995) Specification: $f_t = (m_t - m_t^*) - \phi(q_t - q_t^*)$

[c] RID Model: $f_t = a_0 + a_1(m_t - m_t^*) + a_2(q_t - q_t^*) + a_3(i_t^s - i_t^{s*}) + a_4(i_t^l - i_t^{l*})$

[d] Portfolio Balance Model
$$f_t = a_0 + a_1(m_t - m_t^*) + a_2(q_t - q_t^*) + a_3(i_t^s - i_t^{s*}) + a_4(\pi_t^e - \pi_t^{e*}) + a_5(\overline{TB} - \overline{TB}^*)$$

The null hypothesis: time series has a unit root. [e] the number of lags is determined automatically based on SIC, [f] Newey-West using Bartlett kernel, [g]MacKinnon (1996) one-sided p-values. Constant mean is included in the test equation but no trend.

Table 6. Estimates of Time-Varying MS-ARCH

Parameters	PPP		Mark		RID		H-M	
	Coef.	St. Er.	Coef.	St. Er.	Coef.	St. Er.	Coef.	St. Er.
Austrialian Dollar								
μ_1	-3.051	0.521	-3.073	0.505	-1.119	0.459	-0.703	0.352
μ_2	2.400	0.225	1.586	0.248	0.289	0.269	0.038	0.353
α_1	0.557	0.114	0.359	0.053	0.654	0.238	0.443	0.109
α_2	0.036	0.006	0.068	0.026	0.160	0.111	0.536	0.245
β_1	0.058	0.023	0.125	0.056	0.654	0.264	0.353	0.069
β_2	0.045	0.029	0.345	0.089	0.445	0.321	0.118	0.008
a_0	0.741	0.677	-2.995	2.673	-3.227	1.409	-3.765	1.924
a_1	0.472	0.227	2.546	2.279	-0.384	0.149	-0.255	0.178
a_2	0.155	0.037	-0.549	0.538	0.329	0.168	0.211	0.175
b_0	0.311	0.316	-0.172	0.506	-2.320	1.510	-2.029	1.104
b_1	0.377	0.156	0.425	0.153	-0.180	0.218	-0.195	0.251
b_2	-0.059	0.030	-0.116	0.046	-0.181	0.151	-0.208	0.106
ρ	0.000	2.271	1.000	0.880	1.000	0.156	1.000	0.508
log-likelihood	-436.746		-431.836		-436.498		-434.690	
Canadian dollar								
μ_1	-0.729	0.109	-0.783	0.117	-1.064	0.141	-0.874	0.138
μ_2	1.082	0.336	0.870	0.231	1.080	0.294	1.055	0.251
α_1	0.975	0.432	0.644	0.357	1.085	0.259	0.974	0.323
α_2	0.325	0.134	0.074	0.057	0.454	0.222	0.057	0.027
β_1	-0.006	0.034	0.243	0.089	-0.137	0.075	0.342	0.086
β_2	0.245	0.008	0.537	0.213	0.532	0.223	0.753	0.318
a_0	-0.167	0.962	1.466	0.732	0.693	1.052	1.468	0.904
a_1	3.000	2.115	1.505	0.810	2.046	1.390	1.308	0.752
a_2	0.517	0.367	-0.086	0.043	-0.011	0.100	-0.071	0.057
b_0	-2.318	0.983	-1.274	0.742	-2.060	1.421	-1.467	0.925
b_1	0.322	0.290	0.226	0.199	0.343	0.174	0.333	0.222
b_2	-0.177	0.116	-0.248	0.089	-0.325	0.158	-0.272	0.098
ρ	0.846	89.625	0.000	19.031	0.000	2.733	0.000	2.774
log-likelihood	-338.894		-354.558		-347.222		-354.505	

Note: $\mu=(\mu_1,\mu_2)'$ is the mean of change in the exchange rate, $\alpha=(\alpha_1,\alpha_2)'$ and $\beta=(\beta_1,\beta_2)'$ are intercepts and coefficients from ARCH, $a=(a_0,a_1,a_2)'$ and $b=(b_0,b_1,b_2)'$ are parameters in the transition probabilities, ρ is the unconditional probability of being in state 1 at the initial period

Table 6. Estimates of Time-Varying MS-ARCH (cont'd)

Parameters	**PPP**		**Mark**		**RID**		**H-M**	
	Coef.	St. Er.	Coef.	St. Er.	Coef.	St. Er.	Coef.	St. Er.
Japanese yen								
μ_1	-1.447	0.323	-2.102	0.335	-2.405	0.409	-2.250	0.531
μ_2	1.821	0.483	2.648	0.589	2.575	0.628	6.140	1.286
α_1	0.198	0.052	0.201	0.027	0.829	0.042	0.712	0.135
α_2	0.236	0.044	0.644	0.232	0.554	0.122	0.435	0.212
β_1	0.007	0.006	0.638	0.323	0.016	0.007	0.345	0.086
β_2	0.157	0.068	0.357	0.099	0.854	0.365	1.099	0.444
a_0	-0.127	0.768	0.413	0.503	0.314	0.354	1.272	0.343
a_1	0.154	0.183	0.090	0.056	0.026	0.056	-0.039	0.062
a_2	-0.116	0.044	-0.029	0.011	-0.011	0.017	-0.004	0.017
b_0	1.342	0.401	1.164	0.456	1.053	0.563	0.248	0.722
b_1	-0.027	0.058	-0.095	0.036	-0.076	0.057	-0.088	0.140
b_2	-0.034	0.025	-0.035	0.011	-0.034	0.018	-0.101	0.045
ρ	1.000	0.370	1.000	0.288	1.000	0.498	0.000	0.595
log-likelihood	-479.994		-483.945		-490.923		-473.790	
British pound								
μ_1	-5.223	0.709	-5.121	0.826	-4.467	0.617	-4.495	0.624
μ_2	1.611	0.229	1.592	0.248	1.781	0.252	1.764	0.248
α_1	0.515	0.162	0.424	0.201	0.236	0.044	0.644	0.232
α_2	0.022	0.007	0.345	0.083	0.011	0.061	0.638	0.323
β_1	-0.137	0.075	0.365	0.086	-0.006	0.013	0.234	0.049
β_2	0.532	0.263	0.757	0.318	0.246	0.008	0.535	0.092
a_0	3.572	2.332	3.613	2.257	3.610	1.948	3.713	2.248
a_1	0.621	0.385	0.629	0.375	0.599	0.352	0.616	0.393
a_2	0.003	0.032	-0.012	0.038	0.046	0.043	0.047	0.047
b_0	2.369	0.321	2.391	0.341	2.197	0.358	2.209	0.361
b_1	0.195	0.076	0.192	0.068	0.246	0.075	0.229	0.076
b_2	-0.083	0.040	-0.073	0.039	-0.087	0.035	-0.092	0.036
ρ	1.000	0.923	1.000	0.450	1.000	0.708	1.000	0.496
log-likelihood	-424.162		-424.764		-425.739		-425.373	

Note: $\mu=(\mu_1,\mu_2)'$ is the mean of change in the exchange rate, $\alpha=(\alpha_1,\alpha_2)'$ and $\beta=(\beta_1,\beta_2)'$ are intercepts and coefficients from ARCH, $a=(a_0,a_1,a_2)'$ and $b=(b_0,b_1,b_2)'$ are parameters in the transition probabilities, ρ is the unconditional probability of being in state 1 at the initial period

Table 7. Diagnostic Analysis of ARCH effects in the Exchange Rate

Null Hypothesis:	$\beta 1=\beta 2=0$[a]		$\alpha 1=\alpha 2$, $\beta 1=\beta 2$[b]		**a1=a2=0, b1=b2=0**[c]	
	LR Test[d]	p-value[e]	LR Test	p-value[e]	LR Test	p-value[f]
Australian Dollar						
PPP model	8.055	0.018	6.457	0.040	13.993	0.007
Mark's specification	6.279	0.043	2.678	0.262	4.173	0.383
RID model	19.347	0.000	3.528	0.171	13.497	0.009
H-M model	3.457	0.178	9.146	0.010	9.881	0.042
Canadian Dollar						
PPP model	15.019	0.001	23.568	0.000	10.924	0.027
Mark's specification	7.075	0.029	5.410	0.067	42.252	0.000
RID model	17.541	0.000	19.645	0.000	27.580	0.000
H-M model	2.107	0.349	7.962	0.019	42.146	0.000
Japanese Yen						
PPP model	18.724	0.000	22.581	0.000	14.938	0.005
Mark's specification	8.229	0.016	7.798	0.020	22.840	0.000
RID model	7.519	0.023	26.563	0.000	36.796	0.000
H-M model	6.036	0.049	9.838	0.007	2.530	0.639
British Pound						
PPP model	4.689	0.096	2.744	0.254	12.026	0.017
Mark's specification	2.431	0.297	14.092	0.001	13.230	0.010
RID model	33.276	0.000	2.296	0.317	15.180	0.004
H-M model	8.066	0.018	24.734	0.000	14.448	0.006

a The null hypothesis means that there are no ARCH effects in the conditonal variance, which simply implies the variance is state-dependent but constant over time;
b The null hypothesis means there is no regime shifts in the conditional variance process.
c The null hypothesis means that the transition probability matrix is fixed;
d The Likelihood Ratio is given: $LR = -2(\ln L_R - \ln L_U) \sim \chi^2(J)$, where L_R and L_U are restricted likelihood function and unrestricted likelihood function, and J is the number of restrictions.
e. $\chi^2(2)$ p-value.
f. $\chi^2(4)$ p-value.

Table 8. Forecast Performance of Time-Varying MS-ARCH

	In-Smaple				Out-of-Smaple			
	One-quarter aheand		Two-quarter aheand		One-quarter aheand		Two-quarter aheand	
	MSE-Ratio	p-value	MSE-Ratio	p-value	MSE-Ratio	p-value	MSE-Ratio	p-value
Austrialian Dollar								
PPP	0.894	0.032	0.875	0.024	0.935	0.110	0.914	0.048
Mark	0.997	0.863	1.025	0.538	0.961	3.154	1.206	0.013
RID	0.888	0.025	0.901	0.054	1.064	0.101	0.955	0.208
H-M	0.935	0.073	0.868	0.024	0.939	0.103	1.076	0.099
Canadian dollar								
PPP	0.953	0.504	0.926	0.107	0.921	0.002	0.977	0.332
Mark	0.933	0.085	0.899	0.042	1.000	0.973	1.041	0.353
RID	0.950	0.476	1.003	0.733	0.934	0.081	0.967	0.324
H-M	0.865	0.024	0.899	0.044	0.988	0.853	1.023	0.528
Japanese yen								
PPP	0.942	0.134	0.933	0.074	1.065	0.528	1.092	0.102
Mark	1.084	0.204	0.966	0.645	2.015	0.001	2.427	0.001
RID	0.915	0.053	0.985	0.455	1.123	0.042	2.074	0.000
H-M	0.898	0.033	0.845	0.025	0.974	0.654	0.944	0.105
British pound								
PPP	0.912	0.072	0.876	0.042	1.101	0.214	1.643	0.002
Mark	0.977	0.321	0.943	0.183	1.097	0.321	1.798	0.002
RID	0.923	0.063	0.921	0.058	1.005	0.898	1.254	0.036
H-M	0.899	0.046	0.937	0.087	0.979	0.643	1.065	0.243

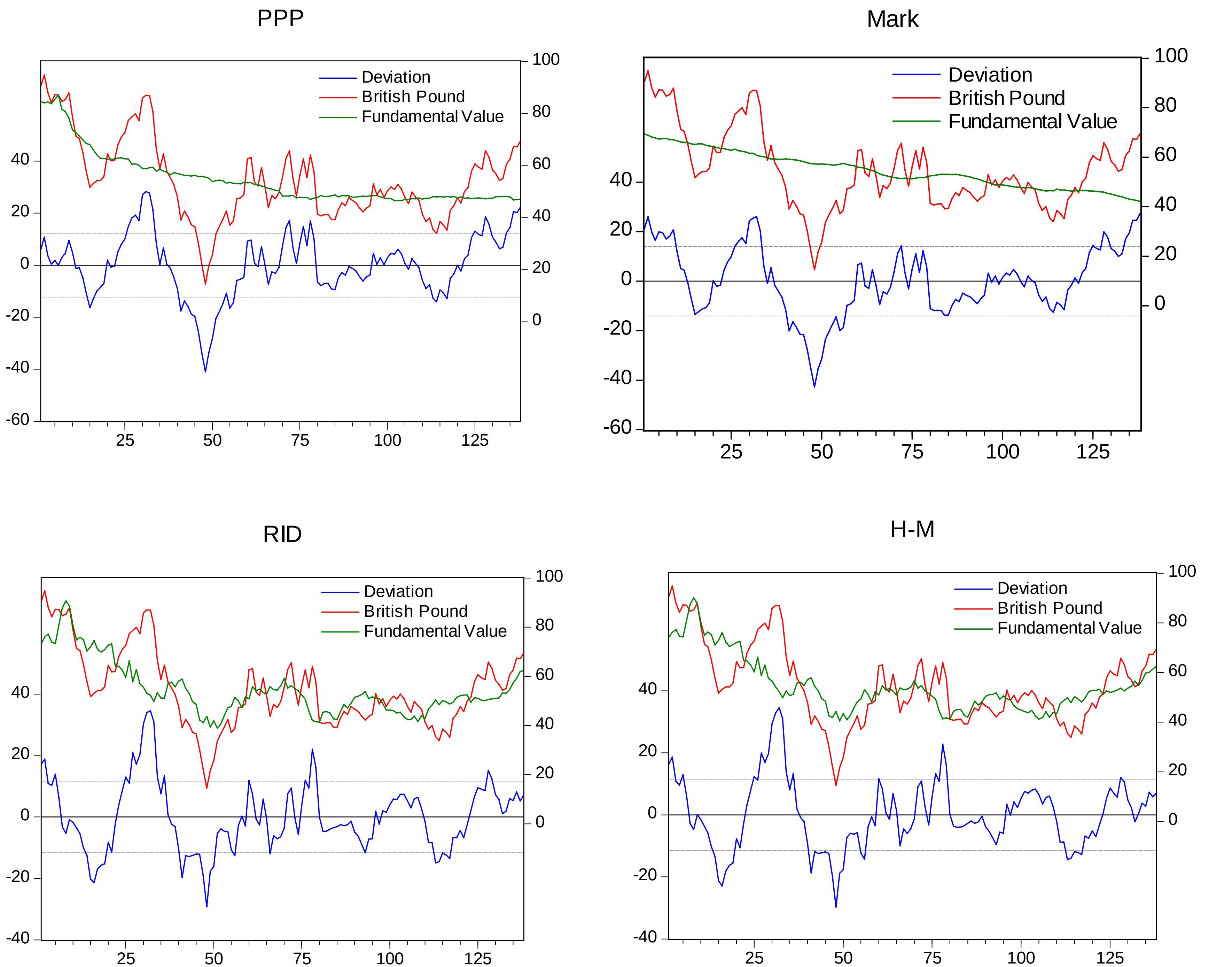
Note: the MSE ratio is the forecast mean squared errors from the relevant model specification relative to that of the random walk specification. The p-value is based on Diebold and Mariano (1995) with the hypothesis that the MSEs from the examined model specification are the same as that of the random walk. Out-of-sample forecasts are computed based on parameters estimated using the sample of 1973:Q1-2000:Q4 and with forecasting periods of 2001:Q1-2007:Q2.

Figure Legends

1. Figure 1. Exchange Rates, Fundamental Values, and Deviations (The British Pound)
2. Figure 2. Conditional Volatility, $\sigma_t^2 = \alpha_{s_t} + \beta_{s_t}(y_{t-1} - \mu_{s_{t-1}})^2$
   (1) The Australian Dollar
   (2) The Canadian Dollar
   (3) The Japanese Yen
   (4) The British Pound
3. Figure 3. Staying Transition Probabilities, Pr($s_t = i \mid s_{t-1} = i$) (The British Pound)
4. Figure 4. Smoothed Probabilities, Pr($s_t = i \mid Y_T$) (The British Pound)

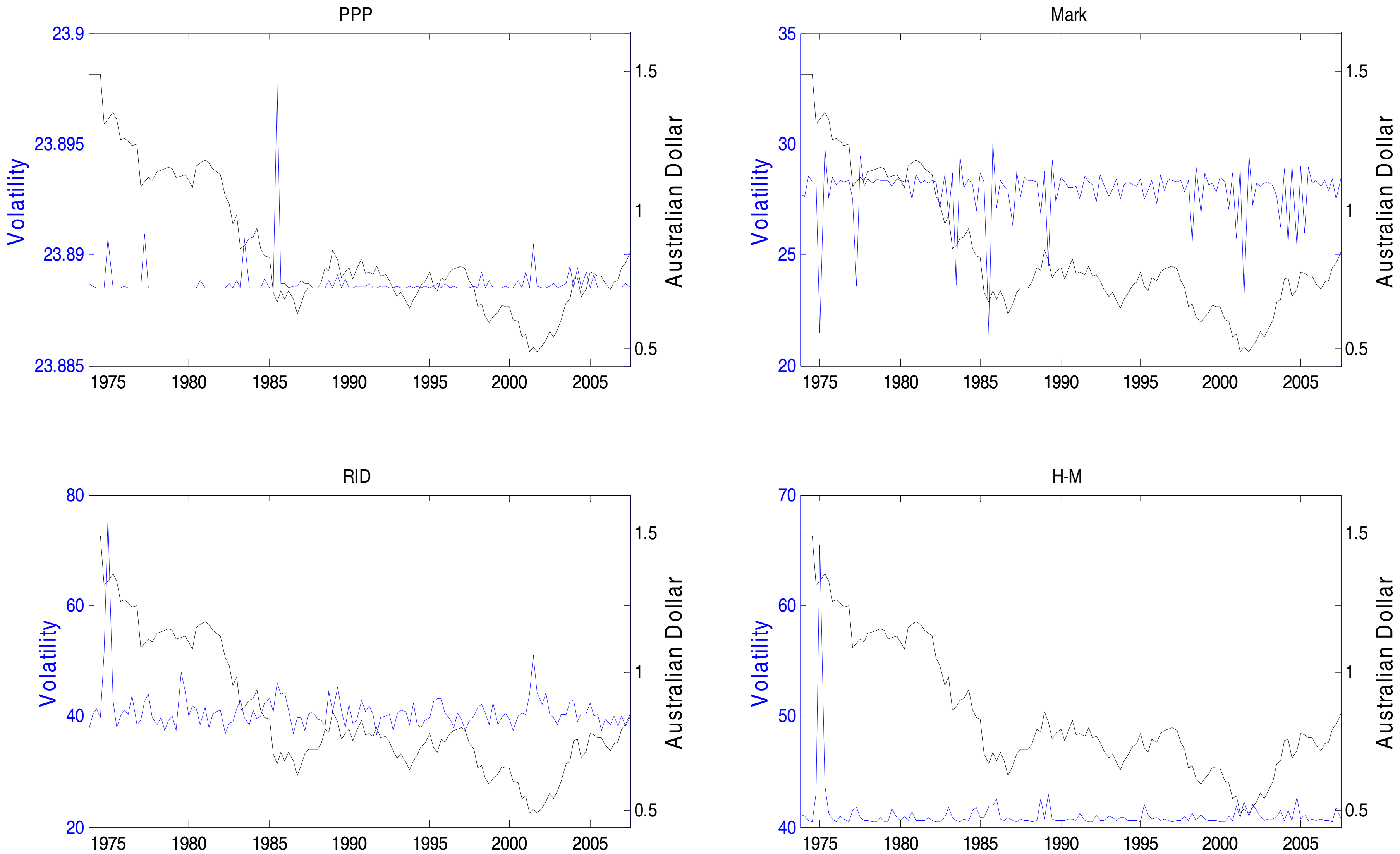
Note that: To save space, Figures 1, 3, and 4 reported herein are only for the case of the British pound. Figures for other currencies (the Australian dollar, the Canadian dollar, and the Japanese yen) are available upon request.

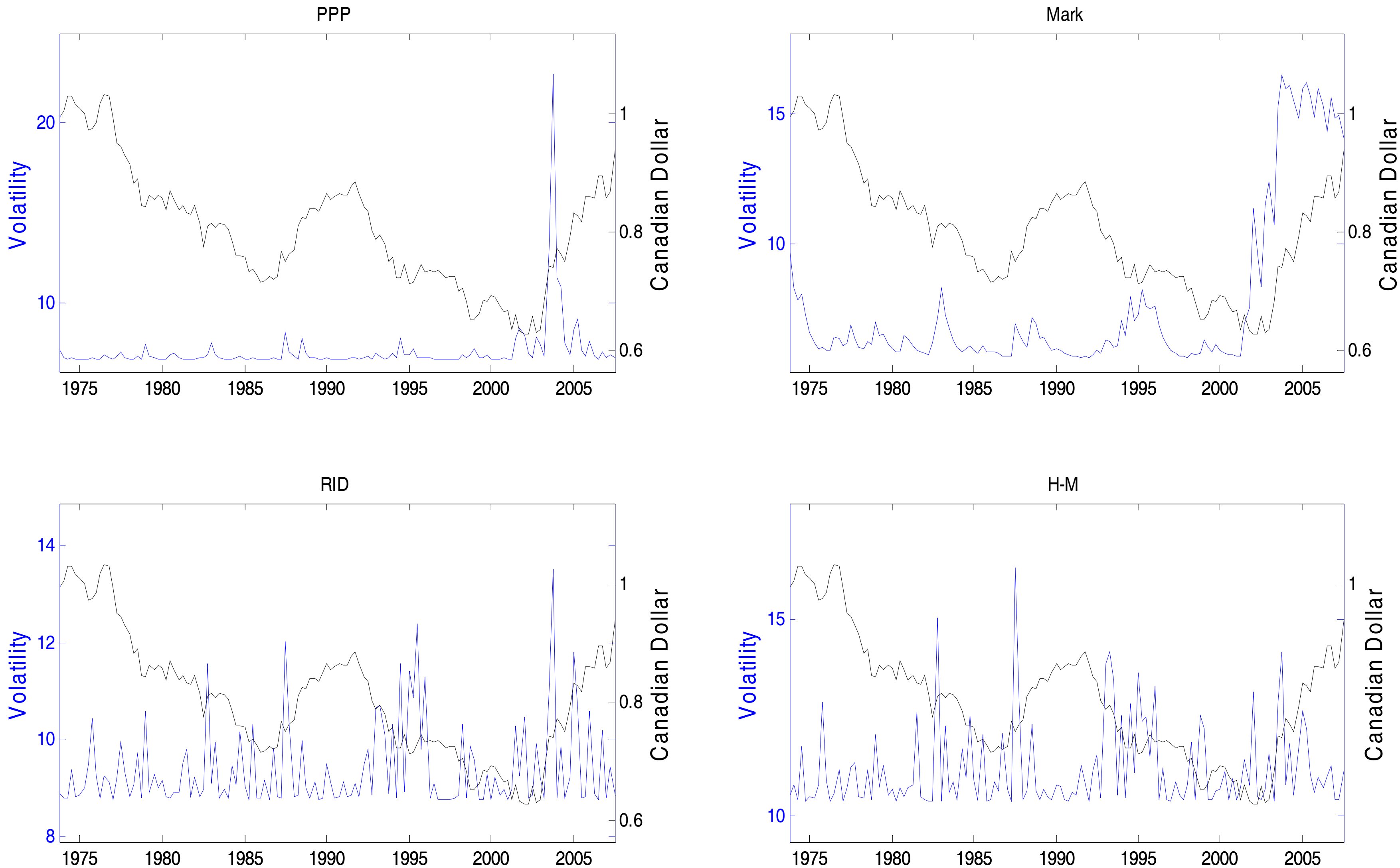
Figure_1

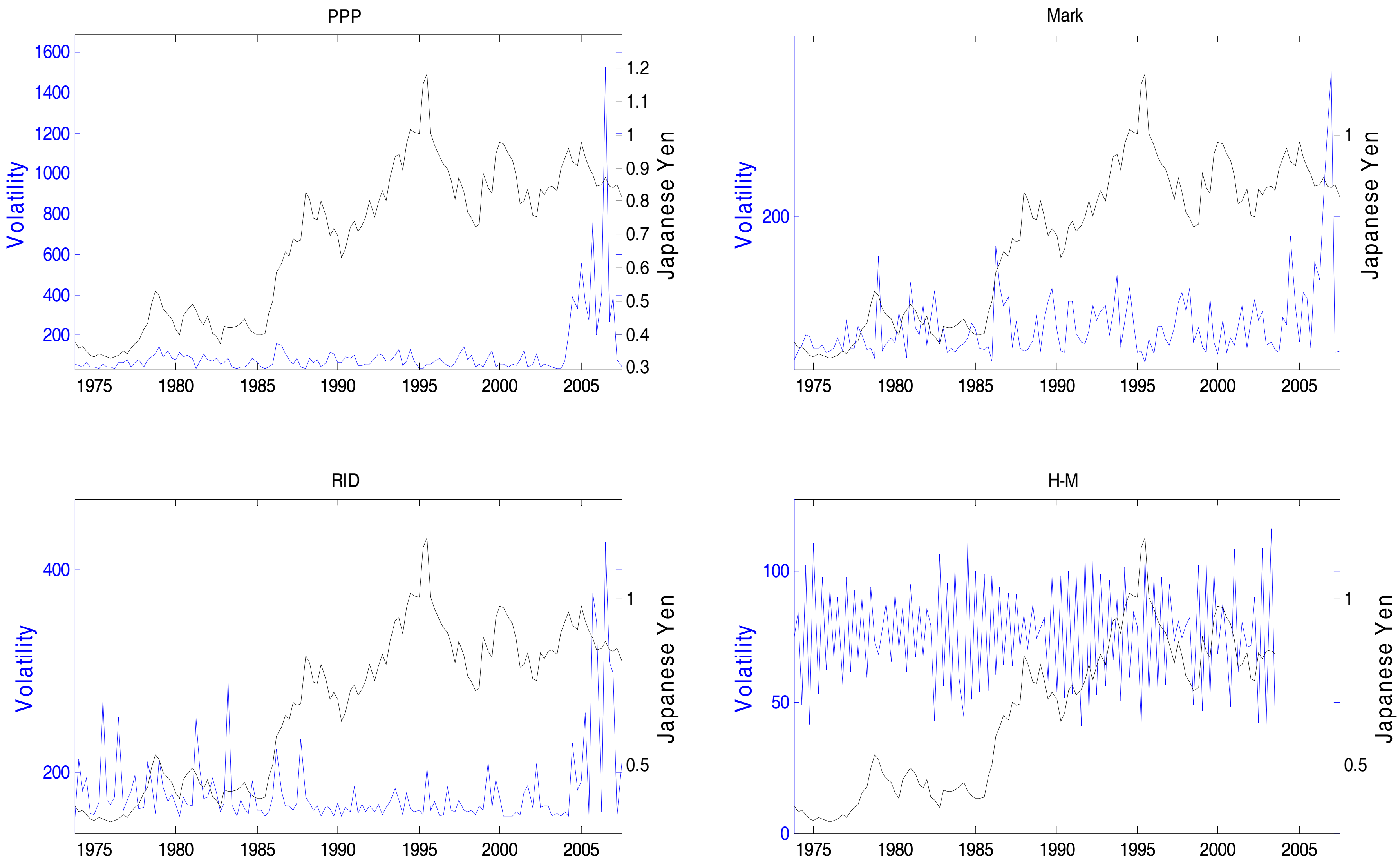


PPP
Deviation
British Pound
Fundamental Value
40
20
0
-20
-40
-60
100
80
60
40
20
0
25
50
75
100
125
Mark
Deviation
British Pound
Fundamental Value
40
20
0
-20
-40
-60
100
80
60
40
20
0
25
50
75
100
125
RID
Deviation
British Pound
Fundamental Value
40
20
0
-20
-40
100
80
60
40
20
0
25
50
75
100
125
H-M
Deviation
British Pound
Fundamental Value
40
20
0
-20
-40
100
80
60
40
20
0
25
50
75
100
125

Figure_2


PPP
23.9
23.895
23.89
23.885
Volatility
1.5
1
0.5
Australian Dollar
1975
1980
1985
1990
1995
2000
2005
Mark
35
30
25
20
Volatility
1.5
1
0.5
Australian Dollar
RID
80
60
40
20
Volatility
1.5
1
0.5
Australian Dollar
H-M
70
60
50
40
Volatility
1.5
1
0.5
Australian Dollar


PPP
Mark
RID
H-M
Volatility
Canadian Dollar
20
10
15
14
12
8
1
0.8
0.6
1975
1980
1985
1990
1995
2000
2005


PPP
Volatility
Japanese Yen
1600
1400
1200
1000
800
600
400
200
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1975
1980
1985
1990
1995
2000
2005
Mark
Volatility
Japanese Yen
200
1
1975
1980
1985
1990
1995
2000
2005
RID
Volatility
Japanese Yen
400
200
1
0.5
1975
1980
1985
1990
1995
2000
2005
H-M
Volatility
Japanese Yen
100
50
0
1
0.5
1975
1980
1985
1990
1995
2000
2005

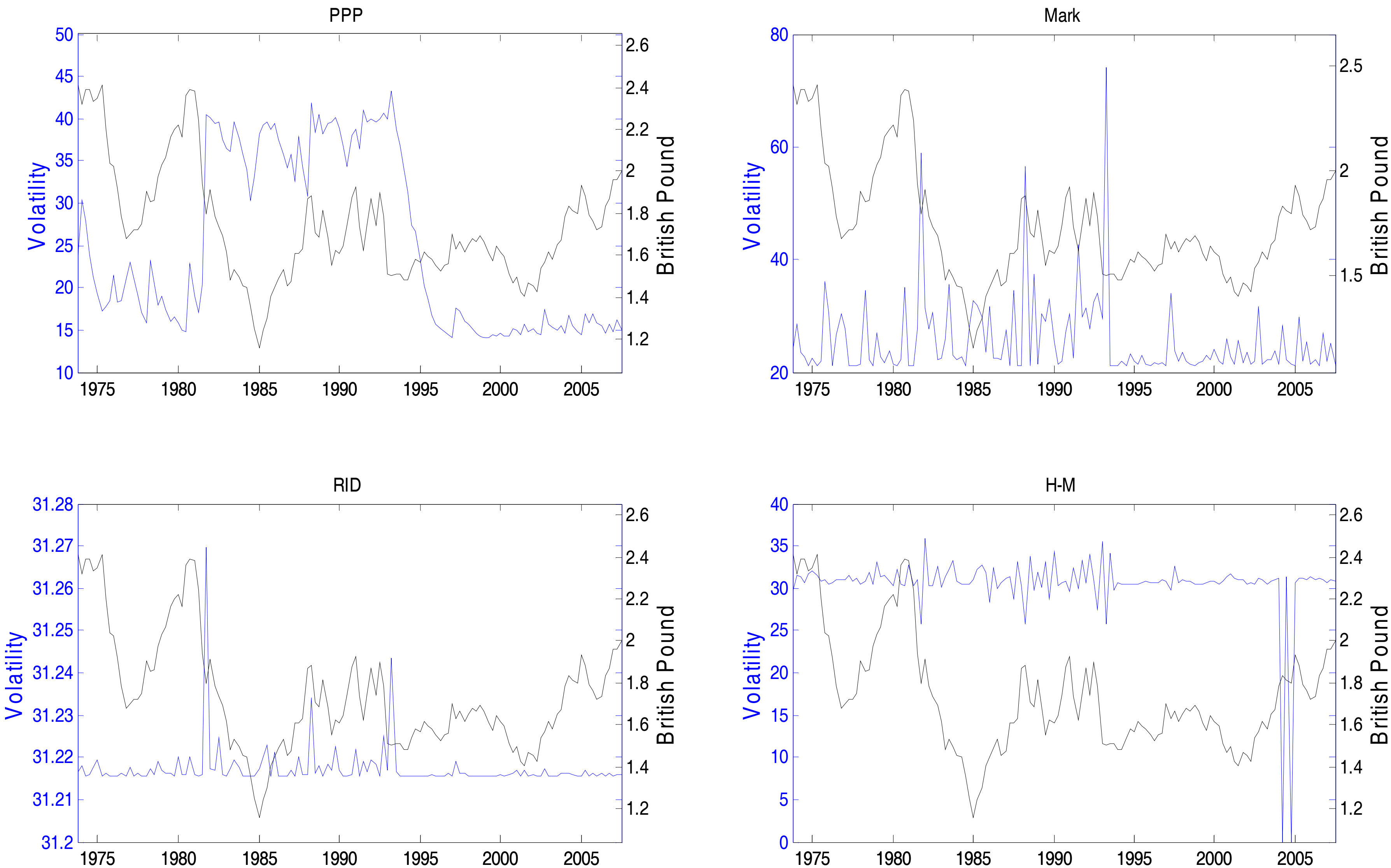

PPP
Mark
RID
H-M
Volatility
British Pound
50
45
40
35
30
25
20
15
10
80
60
40
20
31.28
31.27
31.26
31.25
31.24
31.23
31.22
31.21
31.2
0
5
2.6
2.5
2.4
2.2
2
1.8
1.6
1.5
1.4
1.2
1975
1980
1985
1990
1995
2000
2005

Figure_3

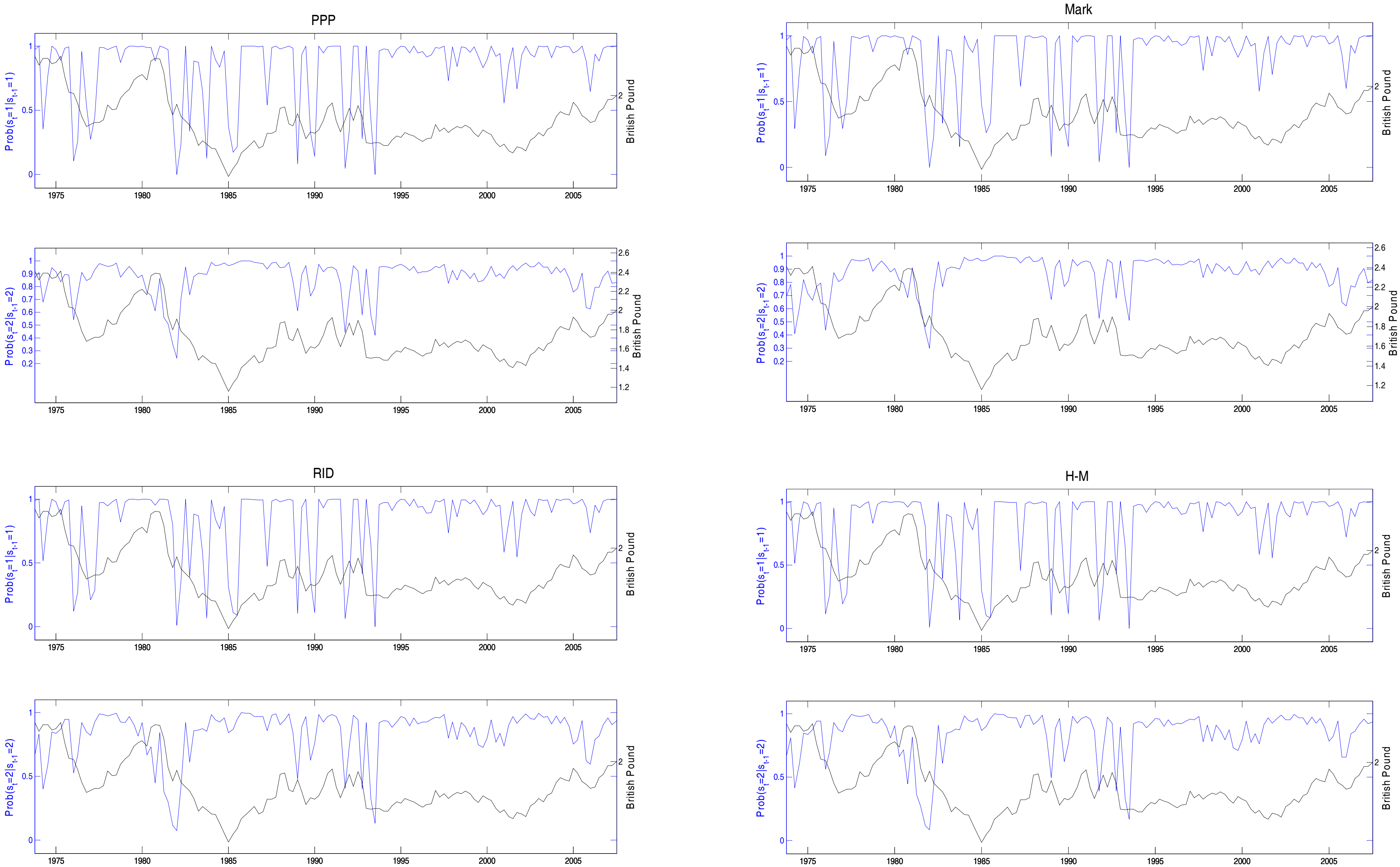
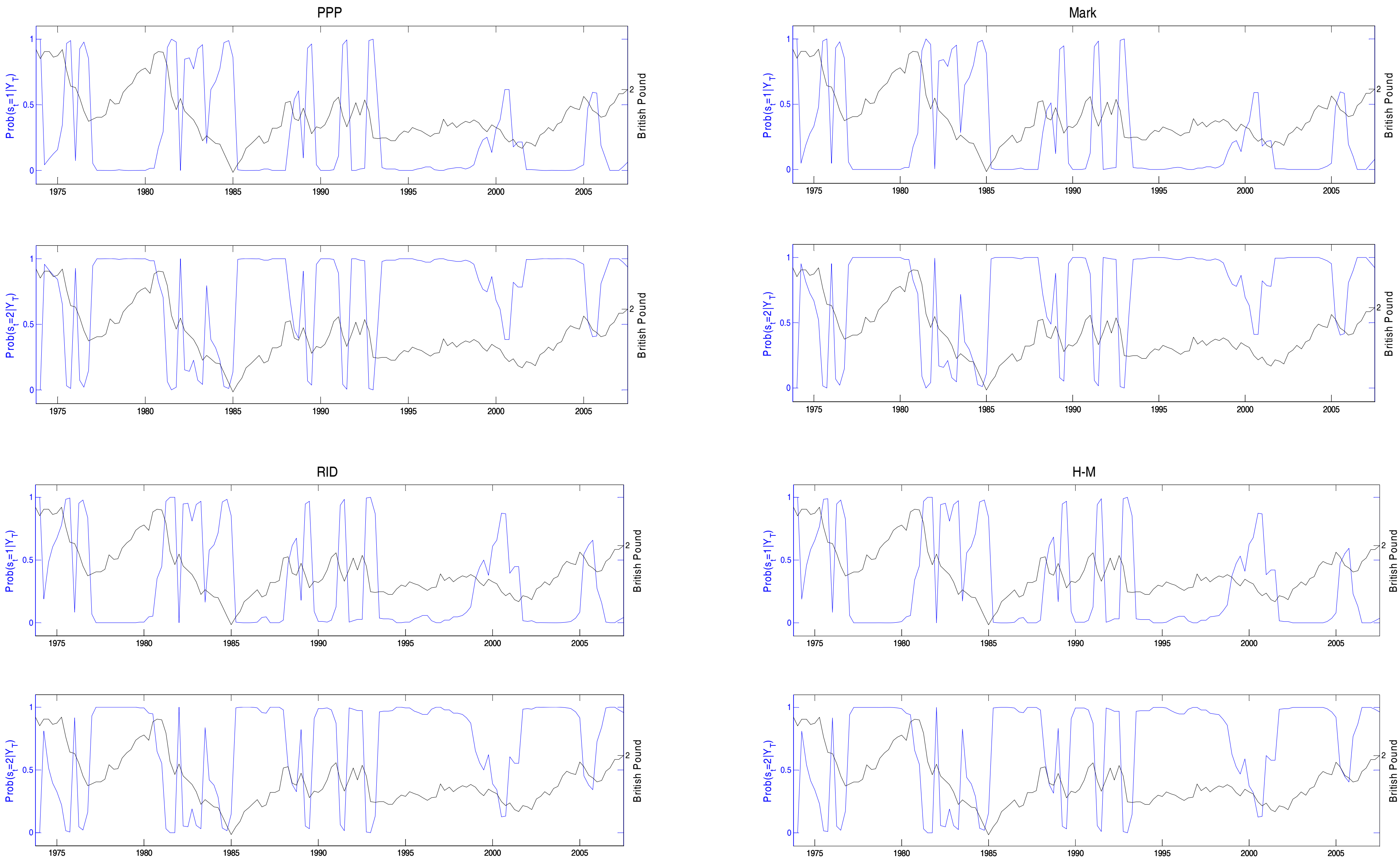

PPP
Mark
RID
H-M
Prob(s_t=1|s_t-1=1)
Prob(s_t=2|s_t-1=2)
British Pound
1975
1980
1985
1990
1995
2000
2005

Figure_4


PPP
Mark
RID
H-M
Prob(s_t=1|Y_T)
Prob(s_t=2|Y_T)
British Pound
1975
1980
1985
1990
1995
2000
2005
0
0.5
1
2