# IS TAX SHARING OPTIMAL?
# AN ANALYSIS IN A PRINCIPAL-AGENT FRAMEWORK

BARNALI GUPTA[1] AND CHRISTELLE VIAUROUX[2]

ABSTRACT. We study the effects of a statutory wage tax sharing rule in a principal - agent framework with moral hazard (à la Holmstrom, 1979) using the approach of Bose, Pal, Sappington (2007) to model the stochastic relationship between the agent's unobserved effort and his observed performance. The analysis indicates that tax sharing with positive legislated contributions from both the employer and employee does not maximize any of the outcomes – employee effort, wages, profits or welfare. Moreover, a rule which specifies a corner solution, with 100% of the tax statutorily levied on the employer will maximize effort, expected profit and expected welfare while 100% of the tax statutorily levied on the employee will maximize expected wages.

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## 1. INTRODUCTION

During the past three decades, the principal-agent framework has become an integral part of the economics literature with incomplete information. (For a survey, see Sappington, 1991, and also Laffont and Martimort, 2002). In this paper, we study the effect of a statutory wage tax sharing rule on wages, effort, profits and aggregate welfare, in a principal agent framework with moral hazard.

Our results show that any interior tax sharing rule with positive legislated contributions from both the employer and employee, will not optimize any outcomes for the principal, agent or the government. Additionally, a statutory rule with 100% of the tax levied on the employer, will maximize effort, expected profit and expected welfare while, under some conditions, a statutory rule of 100% of the tax on the employee will maximize expected wage.

The taxation of wage income in various forms, is common practice and it is equally common to have sharing rules that spilt the tax burden in some fashion between employer and employee. In approximately half of all OECD countries, the shares of employer/employee contributions toward a social security tax, for example, have been stable at approximately 25% of total labor costs. Yet, the distribution of this share between employer/employee varies across countries. There is a 50:50 split in Germany, Switzerland, United States, Luxembourg and Japan. In most other countries, employers typically pay the major share. The exceptions are Denmark and the Netherlands, where employees generally pay the most. This variation and the lack of formal analysis in the literature, motivate the present study.

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*1:*Department of Economics, Miami University, Oxford, Ohio 45056. (513) 529-2856; (513) 529-8047 (fax); Email: guptab@muohio.edu. *2: Corresponding author* Department of Economics, University of Maryland, Baltimore County, Baltimore, MD 21250. Email: ckviauro@umbc.edu.

There is a substantial literature on the optimal income tax in adverse selection models (see Diamond, 1998 and Seade, 1977). The focus of our work is on moral hazard and risk sharing. In the theoretical literature studying the impact of taxes on hours of work, the typical conflict between the substitution effect and income effect has rendered any conclusion logically indeterminate.[1] While this paper is related to the topic of taxation under uncertainty (see Eaton and Rosen 1980 a, b), it is fundamentally different in the model being used and the implication of a statutory tax sharing rule that is being studied. It is useful to see the work by Feldstein (1995) for discussion on the extent to which taxable income as a whole, and not just labor supply, responds to changes in marginal tax rates.

It is clearly illustrated that regardless of the tax distribution chosen as a policy matter, an interior solution with positive shares for both principal and agent, does not optimize any of the relevant outcomes. In fact, in each case, a corner solution does exactly that. From this perspective, it is difficult to justify an interior distribution of tax shares. The next section describes the model. Section 3 presents the analysis while Section 4 concludes and discusses some extensions.

## 2. Model

The model is based on Holmstrom (1979). However, while Holmstrom uses an exponential distribution we adopt the approach of Bose, Pal and Sappington (2007) and represent the stochastic relationship between the agent's effort and the output produced, by a Gamma distribution[2]. [3] An advantage of this distribution is to facilitate the identification of conditions under which the "first-order approach" can be employed to solve the principal's problem (see Jewitt, 1988). The basic framework is the familiar one of a risk neutral employer (or principal) and risk averse employee (or agent) who works for wages. Employee wages are taxed by the government with a statutorily mandated distribution of the tax between the employer and employee. The employer-employee relationship involves moral hazard, where the agent's effort is unobservable by the principal, but it affects the expected outcome as well as the riskiness of outcomes. The realized output is a noisy signal of the agent's effort. Therefore the principal wants to use the contract to induce the agent to exert optimal effort.

More specifically, the principal observes a realized output $x$, $x \in [x_0, \infty)$ and pays the agent an amount $w(x)$. The principal's payoff function is $U^P(x, w) = \pi(x, w) = x - w$. The agent, on the basis of the agreed payment schedule $w(x)$ chooses an action $a$ (effort say) and has a separable von Neumann-Morgenstern utility $U_A(w, a) = 2(w)^\eta - a^2$. We set $\eta = 1/2$ to satisfy the conditions associated to the first order approach to principal-agent problems (Jewitt, 1988)[4]. Note that the usual conditions that $2(w)^\eta$ be increasing concave and $a^2$ be increasing convex are also satisfied. In a different line of business, the agent could receive expected utility $\overline{U}$, so a constraint on the principal's choice of $w$ is that the agent's maximized expected utility must not be less than $\overline{U}$. The technology which is common knowledge is represented by the distribution of output depending on effort, $F(x|a)$ where $F(x|a)$ is absolutely continuous with respect to the same nonnegative measure for each $a$. Hence, $F(x|a)$ has a density $f(x|a)$ . For its flexibility and general properties,

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[1]Eaton and Rosen (1980 a) summarize the extensive econometric research as suggestive of very small responses in hours of work to changes in net wage for prime male earners. However, other groups, such as married women, have considerably higher labor supply response rates.

[2]We use a slightly more general gamma distribution, allowing the minimum output to be strictly positive.

[3]The exponential distribution is a special case of the Gamma distribution, with p = 1.

[4]Note that $\eta \leq \frac{1}{2}$, will satisfy Jewitt's conditions. We use $\eta = 1/2$,to get explicit solutions.

we assume that the output conditional on effort is Gamma distributed $x|a \sim \Gamma(p, a)$ [5], where parameter $p$ is a shape parameter allowing flexibility in the distribution of the output, while effort $a$ simply scales the output distribution horizontally or vertically (see Bose, Pal and Sappington, 2007). We further assume that the government impose a wage tax $t$, of which share $\gamma$ is paid by the employee. Finally, welfare $W(x)$ can be written as $W(x) = x - (1 - t\gamma)w(x) + 2(1 - t\gamma)^{1/2}w(x)^{1/2} - a^2$.

We use the standard principal agent framework with moral hazard. The Principal's problem $[P]$ can be written as:

$$\text{Maximize}_{w,a} \int_{x_0}^{\infty} [x - w(x) - (1 - \gamma)tw(x)] f(x|a, x_0) dx$$

subject to the Participation Constraint

$$\int_{x_0}^{\infty} 2\sqrt{w(x) - t\gamma w(x)} f(x|a, x_0) dx - a^2 = \overline{U}, \tag{2.1}$$

and the Incentive Compatibility Constraint

$$\int_{x_0}^{\infty} 2\sqrt{w(x) - t\gamma w(x)} f_a(x|a, x_0) dx - 2a = 0. \tag{2.2}$$

The following proposition summarizes the results of the above computations. We assume that $\overline{U}$ is sufficiently large such that $w'(x) > 0$ for all $x \geq 0$.

**Proposition 1.** *The solution to the Principal's Problem $[P]$ (second best solution) is characterized by the following set of equations:*

$$\lambda = \frac{1 + t - t\gamma}{2(1 - t\gamma)}(a^2 + \overline{U}), \tag{2.3}$$

$$\mu = \frac{(1 + t - t\gamma)a^3}{p(1 - t\gamma)}, \tag{2.4}$$

$$a = \sqrt[3]{-L + \sqrt{L^2 + K^3}} + \sqrt[3]{-L - \sqrt{L^2 + K^3}}, \tag{2.5}$$

$$w(x) = \frac{1}{4(1 - t\gamma)} \left( \frac{2a}{p}(x - x_0) + \overline{U} - a^2 \right)^2, \tag{2.6}$$

*where*

$$K := \frac{p\overline{U}}{3(p + 4)} \quad \textit{and} \quad L := -\frac{p^2(1 - t\gamma)}{2(p + 4)(1 + t - t\gamma)}. \tag{2.7}$$

Given the solution to $[P]$, we study the relationship between the employee's tax share $\gamma$ and his optimal effort $a$, expected wage $E(w)$, actual wage $w(x)$, the principal's expected profit $E(\pi)$ and the expected aggregate welfare, $E(W)$. These results are presented in Theorem 2 below.

[5] The density function for the gamma distribution is given by: $f(x|a, x_0) = f(x; p, a, x_0) = \frac{1}{a^p \Gamma(p)} (x - x_0)^{p-1} e^{-(x-x_0)/a}$, for $x \in [x_0, \infty)$, where $\Gamma(p) = \int_{x_0}^{\infty} e^{-t} t^{p-1} dt$.

**Theorem 2.** *The following relations hold for all tax shares* $\gamma \in [0, 1]$*:*

*(a)* $\frac{\partial a}{\partial \gamma} < 0$; *i.e.,employee effort is a decreasing function of* $\gamma$.

*(b) If* $1 - t > (2 + \gamma)t$, *then* $\frac{\partial E(w)}{\partial \gamma} > 0$;*i.e. expected wage is an incrasing function of* $\gamma$. *If* $\frac{a^2 - \overline{U}}{2a^2} ap < x - x_0 < ap$, $\frac{\partial w(x)}{\partial \gamma} > 0$*, i.e. for low levels of output, the actual wage is an increasing function of* $\gamma$.

*(c)* $\frac{\partial E(\pi)}{\partial \gamma} < 0$; *i.e. the principal's expected profit is a decreasing function of* $\gamma$.

*(d)* $\frac{\partial E(W)}{\partial \gamma} < 0$; *i.e. expected welfare is a decreasing function of* $\gamma$.

From Theorem 2 (a), we see that the higher the employee's mandated tax share, the lower his effort. Since he is risk averse, ceteris paribus, the agent exerts less effort as his expected post tax wage falls.

If the share of net wages $(1 - t)$, is greater than twice the tax rate $t$, then expected wage increases with $\gamma$ (Theorem 2 (b)). Therefore, if there is an upper bound on the tax rate $t$, we see that as the employee's share of the tax increases, his expected wage will also increase. This is because the risk averse employee has to be compensated with higher expected wage as his share of the wage tax increases. The mandates on tax distribution, place limits on the employer's ability to trade-off risk sharing versus incentives. This result is further substantiated in the second part of theorem 1(b), which shows that the result is valid not only on average but for any given wage, provided that the output is low (between average and its half). If $\overline{U} \geq a^2$, then the conditions imposed in the second part of theorem 1(b) are satisfied provided that the output is smaller than average. Computation results in the next section show that this result is quite robust.)

As the employer's expected wage increases with $\gamma$, so also the principal's expected profit falls with $\gamma$, from Theorem 2 (c). The constraint on the principal's ability to tradeoff risk sharing versus incentives, lowers her expected profit.

Expected welfare is a strictly decreasing function of the agent's share of the wage tax (Theorem 2 (d)). So, while we expect welfare to decrease with a (wage) tax, we see that the decrease in expected profit is larger than the higher expected wage and the expected government revenue from the tax. Hence, while expected wage is maximized if the agent is legislatively mandated to pay 100% of the wage tax, the agent's effort, expected profit and in aggregate, expected welfare, are all minimized. What is abundantly clear is that regardless of the tax distribution chosen as a policy matter, an interior solution with positive shares for both principal and agent, does not optimize any of these outcomes. From this perspective, it is difficult to justify an interior distribution of tax shares.

We explore further, the theoretical findings in Theorem 2 (b) with numerical computation to check the robustness of the sufficient conditions.

## 3. Results

The simulations results below verify the robustness of the solutions from Theorem 2 .

*Conclusion* 1. Employee effort $a$, expected firm profit $E(\pi)$ and expected welfare $E(W)$ are maximized when the statutorily mandated employee's share of the tax $\gamma$ is zero; employee expected wage $E(w)$ is maximized when the statutorily mandated employee's share of the tax is one.

We verify the robustness of our sufficient conditions in Theorem 2 (b)[6] using simulation. Tables 1–2 report the numerical results[7] for $p = 3$, $\overline{U} = 1.5$, $x_0 = 0$, $t$ and $\gamma$ varying from $0.1$ to $0.9$ in increments of $0.1$. We see that given $t$, the expected wage increases with $\gamma$. As stated in the second part of Theorem 2 (b), computation results verify that wage increases with $\gamma$ for a given $t$ for $x = \frac{ap}{2}\left[\frac{a^2-\overline{U}}{2a^2} - 1\right]$ (center of the interval specified).

## 4. CONCLUSION-EXTENSIONS

In the presence of incomplete information, the statutory liability of a tax has very clear implications for effort, profits, wages and aggregate welfare. The theoretical and numerical results do not find any justification for distributing the burden of a wage tax between employer and employee. While the results are derived using specific functions, the point we wish to make is quite general–an interior distribution of tax shares does not maximize any outcomes for any of the parties. Clearly the moral hazard intrinsic in the second best case is critically important to the results obtained here. In the first best case, it can be shown relatively easily that profit and welfare are maximized when the employee's statutory tax share is 100%, while simultaneously wages and tax revenue are minimized.

Finally, we can reasonably wonder whether the agent's reservation utility might depend on the prevailing tax regime. We explored this extension to allow the agent's opportunity wage to depend on the tax environment. For example, if we let $\overline{U} = \overline{U_0}(1 - t\gamma)^\theta$, $(\theta > 0)$ such that $\theta$ is the elasticity of the agent's utility with respect to post tax share of wage, we find sufficient conditions on $\theta$ such that as long as the agent's reservation utility is not "too responsive" to changes in the share of wages that must be paid in taxes, the results from Theorem 2 , with fixed reservation utilty, are generally robust. The conclusions in this paper suggest that further study of the connection between mandated tax liability, its implications for employer and employee earnings and optimal policy in this context, is warranted.

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[6] Robustnmess results of other parts of Theorem 2 are available on request.

[7] Simulations performed for alternative values of p did not affect the results.